

Second Prolog Programming Assignment

Abstract

Task 3: One Move Predicate and a Unit Test

```
-----  
m12 ([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3])  
:-  
Tower1Before=[H|T],  
Tower1After=T,  
Tower2Before=L,  
Tower2After=[H|L].
```

```
test_m12 :-  
write('Testing: move_m12\n'),  
TowersBefore=[[t,s,m,l,h],[],[ ]],  
trace(' ', 'TowersBefore', TowersBefore),  
m12(TowersBefore, TowersAfter),  
trace(' ', 'TowersAfter', TowersAfter).
```

```
?- consult('C:/Users/User/Documents/towers_of_hanoi.pro').  
true.
```

```
?- test_m12.  
Testing: move_m12  
TowersBefore = [[t,s,m,l,h],[],[ ]]  
TowersAfter = [[s,m,l,h],[t],[ ]]  
true.
```

Task 4: Valid State Predicate and Unit Test

```
% -----
m12 ([Tower1Before,Tower2Before,Tower3], [Tower1After,Tower2After,Tower3]) :-
Tower1Before=[H|T],
Tower1After=T,
Tower2Before=L,
Tower2After=[H|L].

% -----
m13 ([Tower1Before,Tower2,Tower3Before], [Tower1After,Tower2,Tower3After]) :-
Tower1Before=[H|T],
Tower1After=T,
Tower3Before=L,
Tower3After=[H|L].

% -----
m21 ([Tower1Before,Tower2Before,Tower3], [Tower1After,Tower2After,Tower3]) :-
Tower2Before=[H|T],
Tower2After=T,
Tower1Before=L,
Tower1After=[H|L].

% -----
m23 ([Tower1,Tower2Before,Tower3Before], [Tower1,Tower2After,Tower3After]) :-
Tower2Before=[H|T],
Tower2After=T,
Tower3Before=L,
Tower3After=[H|L].

% -----
m31 ([Tower1Before,Tower2,Tower3Before], [Tower1After,Tower2,Tower3After]) :-
Tower3Before=[H|T],
Tower3After=T,
Tower1Before=L,
Tower1After=[H|L].

% -----
m32 ([Tower1,Tower2Before,Tower3Before], [Tower1,Tower2After,Tower3After]) :-
Tower3Before=[H|T],
Tower3After=T,
Tower2Before=L,
Tower2After=[H|L].
```

```

% --- Unit test programs

test_m12 :-
write('Testing: move_m12\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m12(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m13 :-
write('Testing: move_m13\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m13(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m21 :-
write('Testing: move_m21\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m21(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m23 :-
write('Testing: move_m23\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m23(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m31 :-
write('Testing: move_m31\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m31(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m32 :-
write('Testing: move_m32\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m32(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

```

```
?- consult('C:/Users/User/Documents/towers_of_hanoi.pro').  
true.
```

```
?- test__m12.  
Testing: move_m12  
TowersBefore = [[t,s,m,l,h],[],[[]]  
TowersAfter = [[s,m,l,h],[t],[[]]  
true.
```

```
?- test__m13.  
Testing: move_m13  
TowersBefore = [[t,s,m,l,h],[],[[]]  
TowersAfter = [[s,m,l,h],[],[t]]  
true.
```

```
?- test__m21.  
Testing: move_m21  
TowersBefore = [[t,s,m,l,h],[],[[]]  
false.
```

```
?- test__m23.  
Testing: move_m23  
TowersBefore = [[t,s,m,l,h],[],[[]]  
false.
```

```
?- test__m31.  
Testing: move_m31  
TowersBefore = [[t,s,m,l,h],[],[[]]  
false.
```

```
?- test__m32.  
Testing: move_m32  
TowersBefore = [[t,s,m,l,h],[],[[]]  
false.
```

Task 5: Valid State Predicate and Unit Test

```
% -----  
% --- valid_state(S) :: S is a valid state  
valid_state([Peg_1,Peg_2,Peg_3]) :-  
valid_peg(Peg_1),valid_peg(Peg_2),valid_peg(Peg_3).  
  
valid_peg([t]).  
valid_peg([t,s]).  
valid_peg([t,m]).  
valid_peg([t,l]).  
valid_peg([t,h]).  
valid_peg([t,s,m,l,h]).  
valid_peg([t,s,m,l]).  
valid_peg([t,s,m,h]).  
valid_peg([t,s,m]).  
  
valid_peg([s]).  
valid_peg([s,m]).  
valid_peg([s,l]).  
valid_peg([s,h]).  
valid_peg([s,m,l,h]).  
valid_peg([s,m,l]).  
valid_peg([s,m,h]).  
  
valid_peg([m]).  
valid_peg([m,l]).  
valid_peg([m,h]).  
valid_peg([m,l,h]).  
  
valid_peg([l]).  
valid_peg([l,h]).  
  
valid_peg([h]).  
valid_peg([]).  
  
?- consult('C:/Users/User/Documents/towers_of_hanoi.pro').  
true.  
  
?- test__valid_state.  
Testing: valid_state  
[[l,t,s,m,h],[],[[]] is invalid.  
[[t,s,m,l,h],[],[[]] is valid.  
[[],[h,t,s,m],[l]] is invalid.  
[[],[t,s,m,h],[l]] is valid.  
[[],[h],[l,m,s,t]] is invalid.  
[[],[h],[t,s,m,l]] is valid.  
true
```

Task 6: Defining the write_sequence predicate

```
124
125 write_solution(S) :-
126 nl, write('Solution ...'), nl, nl,
127 reverse(S,R),
128 write_sequence(R),nl.
129
130 write_sequence([]).
131
132 write_sequence([H|T]) :-
133 elaborate(H,E),
134 write(E),nl,
135 write_sequence(T).
136
137 elaborate(m12,Elaboration) :-
138 Elaboration='Transfer A Disk From Tower 1 to Tower 2.'.
139
140 elaborate(m13,Elaboration) :-
141 Elaboration='Transfer A Disk From Tower 1 to Tower 3.'.
142
143 elaborate(m21,Elaboration) :-
144 Elaboration='Transfer A Disk From Tower 2 to Tower 1.'.
145
146 elaborate(m23,Elaboration) :-
147 Elaboration='Transfer A Disk From Tower 2 to Tower 3.'.
148
149 elaborate(m31,Elaboration) :-
150 Elaboration='Transfer A Disk From Tower 3 to Tower 1.'.
151
152 elaborate(m32,Elaboration) :-
153 Elaboration='Transfer A Disk From Tower 3 to Tower 2.'.
154
...

?- consult('C:/Users/User/Documents/towers_of_hanoi.pro').
true.

?- test_write_sequence.
First test of write_sequence ...
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Second test of write_sequence ...
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 2.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 2 to Tower 3.
Transfer A Disk From Tower 1 to Tower 3.
true.

?- ■
```

Task 7: Run the Program to Solve the 3 Disk Problem

```
?- consult('C:/Users/User/Documents/towers_of_hanoi.pro').
true.

?- solve.
PathSoFar = [[[s,m,1],[],[[]]]]
Move = m12
NextState = [[m,1],[s],[[]]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]]]
Move = m12
NextState = [[1],[m,s],[[]]]
Move = m13
NextState = [[1],[s],[m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]]]
Move = m12
NextState = [[],[1,s],[m]]
Move = m13
NextState = [[],[s],[1,m]]
Move = m21
NextState = [[s,1],[],[m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]]]
Move = m12
NextState = [[1],[s],[m]]
Move = m13
NextState = [[1],[],[s,m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]]]
Move = m12
NextState = [[],[1],[s,m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[1],[1],[s,m]]]
Move = m21
NextState = [[1],[],[s,m]]
Move = m23
NextState = [[],[],[1,s,m]]
Move = m31
NextState = [[s],[1],[m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[1],[1],[s,m]],[[s],[1],[m]]]
Move = m12
NextState = [[],[s,1],[m]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[1],[1],[s,m]],[[s],[1],[m]],[[1],[s,1],[m]]]
Move = m21
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[1],[s,m]]
Move = m31
NextState = [[m],[s,1],[[]]]
PathSoFar = [[[s,m,1],[],[[]]],[[m,1],[s],[[]]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[1],[1],[s,m]],[[s],[1],[m]],[[1],[s,1],[m]],[[m],[s,1],[[]]]]
Move = m12
```

```

Move = m12
NextState = [[],[m,s,l],[ ]]
Move = m13
NextState = [[],[s,l],[m]]
Move = m21
NextState = [[s,m],[l],[ ]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]]]]
Move = m12
NextState = [[m],[s,l],[ ]]
Move = m13
NextState = [[m],[l],[s]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]]]]]
Move = m12
NextState = [[],[m,l],[s]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]],[[ ],[m,l],[s]]]]]
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[ ]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]],[[ ],[m,l],[s]],[[s],[m,l],[ ]]]]
Move = m12
NextState = [[],[s,m,l],[ ]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]],[[ ],[m,l],[s]],[[s],[m,l],[ ]],[[ ],[s,m,l],[ ]]]]
Move = m21
NextState = [[s],[m,l],[ ]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[ ]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[ ]]
PathSoFar = [[[s,m,l],[ ],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ],[s,m]],[[ ],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]],[[ ],[m,l],[s]],[[ ],[s,m,l],[ ]]]]
Move = m21

```



```

Move = m21
NextState = [[s],[m,l],[ ]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[s,m]],[[s],[l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[m],[l],[s]],[[ ],[m,l],[s]],[[ ],[s,m,l],[ ]],[[s],[m,l],[ ]]]]
Move = m12
NextState = [[],[s,m,l],[ ]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[ ]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[l],[m,s]]
Move = m21
NextState = [[l,m],[ ],[s]]
Move = m23
NextState = [[m],[ ],[l,s]]
Move = m31
NextState = [[s,m],[l],[ ]]
Move = m32
NextState = [[m],[s,l],[ ]]
Move = m21
NextState = [[l,s,m],[ ],[ ]]
Move = m23
NextState = [[s,m],[ ],[l]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[s,m],[ ],[l]]]]]
Move = m12
NextState = [[m],[s],[l]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[s,m],[ ],[l]],[[m],[s],[l]]],[[ ],[s],[m,l]]]]]
Move = m12
NextState = [[],[m,s],[l]]
Move = m13
NextState = [[],[s],[m,l]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[s,m],[ ],[l]],[[m],[s],[l]],[[ ],[s],[m,l]]]]]
Move = m21
NextState = [[s],[ ],[m,l]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[s,m],[ ],[l]],[[m],[s],[l]],[[ ],[s],[m,l]],[[s],[ ],[m,l]]]]]
Move = m12
NextState = [[],[s],[m,l]]
Move = m12
NextState = [[],[s],[m,l]]
Move = m13
NextState = [[],[ ],[s,m,l]]
PathSoFar = [[[s,m,l],[ ]],[ ]],[[m,l],[s],[ ]],[[l],[s],[m]],[[s,l],[ ],[m]],[[l],[ ]],[s,m]],[[l],[l],[s,m]],[[s],[l],[m]],[[ ],[s,l],[m]],[[m],[s,l],[ ]],[[s,m],[l],[ ]],[[s,m],[ ],[l]],[[m],[s],[l]],[[ ],[s],[m,l]],[[s],[ ],[m,l]]]]]
SolutionSoFar = [m12,m13,m21,m13,m12,m31,m12,m31,m21,m23,m12,m13,m21,m13]

```

Solution ...

```
Transfer A Disk From Tower 1 to Tower 2.  
Transfer A Disk From Tower 1 to Tower 3.  
Transfer A Disk From Tower 2 to Tower 1.  
Transfer A Disk From Tower 1 to Tower 3.  
Transfer A Disk From Tower 1 to Tower 2.  
Transfer A Disk From Tower 3 to Tower 1.  
Transfer A Disk From Tower 1 to Tower 2.  
Transfer A Disk From Tower 3 to Tower 1.  
Transfer A Disk From Tower 2 to Tower 1.  
Transfer A Disk From Tower 2 to Tower 3.  
Transfer A Disk From Tower 1 to Tower 2.  
Transfer A Disk From Tower 1 to Tower 3.  
Transfer A Disk From Tower 2 to Tower 1.  
Transfer A Disk From Tower 1 to Tower 3.
```

true

1. 14 moves
2. 7 moves
3. The program was constructed to test all possible paths, checking the validity of all moves and then making the corresponding move that the program traced thru.

Task 8: Run the Program to Solve the 4 Disk Problem

Solution ...

```
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 3.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 2 to Tower 1.
Transfer A Disk From Tower 1 to Tower 3.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 3 to Tower 1.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 2 to Tower 3.
Transfer A Disk From Tower 1 to Tower 2.
Transfer A Disk From Tower 2 to Tower 3.
```

1. 35 moves
2. 15 moves

Task 9: Review Your Code and Archive It

```
% -----
% -----
% --- File: towers_of_hanoi.pro
% --- Line: Program to solve the Towers of Hanoi problem
% -----
:- consult('C:/Users/User/Documents/Saved to this PC/pro').
% -----
% --- make_move(S,T,SSO) :: Make a move from state S to state T by SSO
make_move(TowersBeforeMove,TowersAfterMove,m12) :-
m12(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m13) :-
m13(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m21) :-
m21(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m23) :-
m23(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m31) :-
m31(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m32) :-
m32(TowersBeforeMove,TowersAfterMove).

% -----
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower1Before=[H|T],
Tower1After=T,
Tower2Before=L,
Tower2After=[H|L].

% -----
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
Tower1Before=[H|T],
Tower1After=T,
Tower3Before=L,
Tower3After=[H|L].

% -----
```

```

% -----
m21 ([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-
Tower2Before=[H|T],
Tower2After=T,
Tower1Before=L,
Tower1After=[H|L].

% -----
m23 ([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-
Tower2Before=[H|T],
Tower2After=T,
Tower3Before=L,
Tower3After=[H|L].

% -----
m31 ([Tower1Before, Tower2, Tower3Before], [Tower1After, Tower2, Tower3After]) :-
Tower3Before=[H|T],
Tower3After=T,
Tower1Before=L,
Tower1After=[H|L].

% -----
m32 ([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-
Tower3Before=[H|T],
Tower3After=T,
Tower2Before=L,
Tower2After=[H|L].

% -----
% --- valid_state(S) :: S is a valid state
valid_state ([Peg_1, Peg_2, Peg_3]) :-
valid_peg (Peg_1), valid_peg (Peg_2), valid_peg (Peg_3).

valid_peg ([t]).
valid_peg ([t, s]).
valid_peg ([t, m]).
valid_peg ([t, l]).

```

```

% -----
% --- valid_state(S) :: S is a valid state
valid_state([Peg_1,Peg_2,Peg_3]) :-
valid_peg(Peg_1),valid_peg(Peg_2),valid_peg(Peg_3).

valid_peg([t]).
valid_peg([t,s]).
valid_peg([t,m]).
valid_peg([t,l]).
valid_peg([t,h]).
valid_peg([t,s,m,l,h]).
valid_peg([t,s,m,l]).
valid_peg([t,s,m,h]).
valid_peg([t,s,m]).

valid_peg([s]).
valid_peg([s,m]).
valid_peg([s,l]).
valid_peg([s,h]).
valid_peg([s,m,l,h]).
valid_peg([s,m,l]).
valid_peg([s,m,h]).

valid_peg([m]).
valid_peg([m,l]).
valid_peg([m,h]).
valid_peg([m,l,h]).

valid_peg([l]).
valid_peg([l,h]).

valid_peg([h]).
valid_peg([]).

```

```

% -----
% --- solve(Start,Solution) :: succeeds if Solution represents a path
% --- from the start state to the goal state.
solve :-
  extend_path([[s,m,l],[],[[]],[[]],[[]],Solution),
  write_solution(Solution).
extend_path(PathSoFar,SolutionSoFar,Solution) :-
  PathSoFar = [[[]],[[]],[s,m,l]|_],
  showr('PathSoFar',PathSoFar),
  showr('SolutionSoFar',SolutionSoFar),
  Solution = SolutionSoFar.
extend_path(PathSoFar,SolutionSoFar,Solution) :-
  PathSoFar = [CurrentState|_],
  showr('PathSoFar',PathSoFar),
  make_move(CurrentState,NextState,Move),
  show('Move',Move),
  show('NextState',NextState),
  not(member(NextState,PathSoFar)),
  valid_state(NextState),
  Path = [NextState|PathSoFar],
  Soln = [Move|SolutionSoFar],
  extend_path(Path,Soln,Solution).

% -----
% --- write_sequence_reversed(S) :: Write the sequence, given by S,
% --- expanding the tokens into meaningful strings.

write_solution(S) :-
  nl, write('Solution ...'), nl, nl,
  reverse(S,R),
  write_sequence(R),nl.

write_sequence([]).

write_sequence([H|T]) :-
  elaborate(H,E),
  write(E),nl,

```

```

write_sequence([H|T]) :-
elaborate(H,E),
write(E),nl,
write_sequence(T).

elaborate(m12,Elaboration) :-
Elaboration='Transfer A Disk From Tower 1 to Tower 2.'.

elaborate(m13,Elaboration) :-
Elaboration='Transfer A Disk From Tower 1 to Tower 3.'.

elaborate(m21,Elaboration) :-
Elaboration='Transfer A Disk From Tower 2 to Tower 1.'.

elaborate(m23,Elaboration) :-
Elaboration='Transfer A Disk From Tower 2 to Tower 3.'.
█
elaborate(m31,Elaboration) :-
Elaboration='Transfer A Disk From Tower 3 to Tower 1.'.

elaborate(m32,Elaboration) :-
Elaboration='Transfer A Disk From Tower 3 to Tower 2.'.

% -----
% ---Unit test program

test_valid_state :-
write('Testing: valid_state\n'),
test_vs([[l,t,s,m,h],[],[[]]),
test_vs([[t,s,m,l,h],[],[[]]),
test_vs([[],[h,t,s,m],[l]]),
test_vs([[],[t,s,m,h],[l]]),
test_vs([[],[h],[l,m,s,t]]),
test_vs([[],[h],[t,s,m,l]]).

test_vs(S) :-

```



```

test_vs(S) :-
valid_state(S),
write(S), write(' is valid. '), nl.
test_vs(S) :-
write(S), write(' is invalid. '), nl.

% -----
% --- Unit test programs

test_m12 :-
write('Testing: move_m12\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m12(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m13 :-
write('Testing: move_m13\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m13(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m21 :-
write('Testing: move_m21\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m21(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m23 :-
write('Testing: move_m23\n'),
TowersBefore=[[t,s,m,l,h],[],[ ]],
trace('','TowersBefore',TowersBefore),
m23(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

```

```

test_m31 :-
write('Testing: move_m31\n'),
TowersBefore=[t,s,m,l,h],[],[],
trace('','TowersBefore',TowersBefore),
m31(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).

test_m32 :-
write('Testing: move_m32\n'),
TowersBefore=[t,s,m,l,h],[],[],
trace('','TowersBefore',TowersBefore),
m32(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).
■
% -----
% --- Unit Test Program

test_write_sequence :-
write('First test of write_sequence ...'), nl,
write_sequence([m31,m12,m13,m21]),
write('Second test of write_sequence ...'), nl,
write_sequence([m13,m12,m32,m13,m21,m23,m13]).

```