
Second Prolog Programming Assignment Solution

Task 3: One Move Predicate and a Unit Test

```
m12([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].
```

Unit Test Demo

```
?- test_m12.  
Testing: move_m12  
TowersBefore = [[t,s,m,l,h],[ ],[]]  
TowersAfter = [[s,m,l,h],[t],[ ]]  
true.
```

Task 4: The Remaining Five Move Predicates and a Unit Tests

```
m12([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].  
m13([Tower1Before, Tower2Before, Tower3Before], [Tower1After, Tower2After, Tower3After]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower3Before = [H|L].  
m21([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
m23([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower3Before = L,  
    Tower3After = [H|L].  
m31([Tower1Before, Tower2Before, Tower3Before], [Tower1After, Tower2After, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
m32([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].
```

*

Unit Test Demo

```

?- test_m12.
Testing: move_m12
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[t],[]]
true.

?- test_m13.
Testing: move_m13
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[],[t]]
true.

```

```

?- test_m21.
Testing: move_m21
TowersBefore = [[], [t,s,m,l,h], []]
TowersAfter = [[t], [s,m,l,h], []]
true.

?- test_m23.
Testing: move_m23
TowersBefore = [[], [t,s,m,l,h], []]
TowersAfter = [[], [s,m,l,h], [t]]
true.

```

```

?- test_m31.
Testing: move_m31
TowersBefore = [[],[],[t,s,m,l,h]]
TowersAfter = [[t],[],[s,m,l,h]]
true.

?- test_m32.
Testing: move_m32
TowersBefore = [[],[],[t,s,m,l,h]]
TowersAfter = [[],[],[s,m,l,h]]
true.

```

Task 5: Valid State Predicate and Unit Test

```

valid_state([Tower1,Tower2,Tower3]) :-  

    valid_tower(Tower1),  

    valid_tower(Tower2),  

    valid_tower(Tower3).  
  

valid_tower([]).  
  

valid_tower([t]).  

valid_tower([s]).  

valid_tower([m]).  

valid_tower([l]).  

valid_tower([h]).  
  

valid_tower([t, s]).  

valid_tower([t, m]).  

valid_tower([t, l]).  

valid_tower([t, h]).  
  

valid_tower([s, m]).  

valid_tower([s, l]).  

valid_tower([s, h]).  
  

valid_tower([m, h]).  

valid_tower([m, l]).  
  

valid_tower([l, h]).  
  

valid_tower([t, s, h]).  

valid_tower([t, s, l]).  

valid_tower([t, s, m]).  

valid_tower([t, m, h]).  

valid_tower([t, m, l]).  
  

valid_tower([s, m, l]).  

valid_tower([s, m, h]).  

valid_tower([s, l, h]).  
  

valid_tower([m, l, h]).  
  

valid_tower([t, s, l, h]).  

valid_tower([t, m, l, h]).  

valid_tower([t, s, m, h]).  

valid_tower([t, s, m, l]).  
  

valid_tower([s, m, l, h]).  

valid_tower([t, s, m, l, h]).  


```

Unit Test Program Demo

```
?- test_valid_state.  
Testing: valid_state  
[[l,t,s,m,h],[[],[]]] is invalid.  
[[t,s,m,l,h],[[],[]]] is valid.  
[[],[h,t,s,m],[l]] is invalid.  
[[],[t,s,m,h],[l]] is valid.  
[[],[h],[l,m,s,t]] is invalid.  
[[],[h],[t,s,m,l]] is valid.  
true
```

Task 6: Defining the write sequence predicate

```
write_solution(S) :-  
    nl, write('Solution ...'), nl, nl, reverse(S,R), write_sequence(R), nl.  
write_sequence([]).  
write_sequence([H|T]) :-  
    elaborate(H, E), write(E), nl, write_sequence(T).  
  
elaborate(m12, E) :-  
    E = "Move the top Disk from tower 1 to tower 2.".  
elaborate(m13, E) :-  
    E = "Move the top Disk from tower 1 to tower 3.".  
elaborate(m21, E) :-  
    E = "Move the top Disk from tower 2 to tower 1.".  
elaborate(m23, E) :-  
    E = "Move the top Disk from tower 2 to tower 3.".  
elaborate(m31, E) :-  
    E = "Move the top Disk from tower 3 to tower 1.".  
elaborate(m32, E) :-  
    E = "Move the top Disk from tower 3 to tower 2.".
```

Unit Test Program Demo

```
?- test_write_sequence.  
First test of write_sequence ...  
Move the top Disk from tower 3 to tower 1  
Move the top Disk from tower 1 to tower 2  
Move the top Disk from tower 1 to tower 3  
Move the top Disk from tower 2 to tower 1  
Second test of write_sequence ...  
Move the top Disk from tower 1 to tower 3  
Move the top Disk from tower 1 to tower 2  
Move the top Disk from tower 3 to tower 2  
Move the top Disk from tower 1 to tower 3  
Move the top Disk from tower 2 to tower 1  
Move the top Disk from tower 2 to tower 3  
Move the top Disk from tower 1 to tower 3  
true.
```

Task 7: Run the program to solve the 3 disk problem

?- solve.

PathSoFar = [[[s,m,l],[[],[]]]]

Move = m12

NextState = [[m,l],[s,[]]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]]]]]

Move = m12

NextState = [[l],[m,s,[]]]

Move = m13

NextState = [[l],[s],[m]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]],[[l],[s],[m]]]]]

Move = m12

NextState = [[],[l,s],[m]]

Move = m13

NextState = [[],[s],[l,m]]

Move = m21

NextState = [[s,l],[[],[m]]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]],[[l],[s],[m]],[[s,l],[[],[m]]]]]]

Move = m12

NextState = [[l],[s],[m]]

Move = m13

NextState = [[l],[[],[s,m]]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]],[[l],[s],[m]],[[s,l],[[],[m]],[[l],[[],[s,m]]]]]]

Move = m12

NextState = [[],[l],[s,m]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]],[[l],[s],[m]],[[s,l],[[],[m]],[[l],[[],[s,m]]],[[l],[[],[s,m]]]]]]

Move = m21

NextState = [[l],[[],[s,m]]]

Move = m23

NextState = [[],[[],[l,s,m]]]

Move = m31

NextState = [[s],[l],[m]]

PathSoFar = [[[s,m,l],[[],[]],[[m,l],[s,[]],[[l],[s],[m]],[[s,l],[[],[m]],[[l],[[],[s,m]]],[[l],[[],[s,m]]],[[s],[l],[m]]]]]

Move = m12

NextState = [[], [s], [l], [m]]

PathSoFar = [[[s, m, l], [], []], [[m, l], [s], []], [[l], [s], [m]], [[s, l], [], [m]], [[l], [], [s, m]], [[], [l], [s, m]], [[s], [l], [m]], [[], [s, l], [m]]]]

Move = m21

NextState = [[s], [l], [m]]

Move = m23

NextState = [[], [l], [s, m]]

Move = m31

NextState = [[m], [s, l], []]

PathSoFar =

[[[s, m, l], [], []], [[m, l], [s], []], [[l], [s], [m]], [[s, l], [], [m]], [[l], [], [s, m]], [[], [l], [s, m]], [[s], [l], [m]], [[], [s, l], [m]], [[m], [s, l], []]]]

Move = m12

NextState = [[], [m, s, l], []]

Move = m13

NextState = [[], [s, l], [m]]

Move = m21

NextState = [[s, m], [l], []]

PathSoFar =

[[[s, m, l], [], []], [[m, l], [s], []], [[l], [s], [m]], [[s, l], [], [m]], [[l], [], [s, m]], [[], [l], [s, m]], [[s], [l], [m]], [[], [s, l], [m]], [[m], [s, l], []], [[s, m], [l], []], [[m], [l], [s]]]]

Move = m12

NextState = [[m], [s, l], []]

Move = m13

NextState = [[m], [l], [s]]

PathSoFar =

[[[s, m, l], [], []], [[m, l], [s], []], [[l], [s], [m]], [[s, l], [], [m]], [[l], [], [s, m]], [[], [l], [s, m]], [[s], [l], [m]], [[], [s, l], [m]], [[m], [s, l], []], [[s, m], [l], []], [[m], [l], [s]]]]

Move = m12

NextState = [[], [m, l], [s]]

PathSoFar =

[[[s, m, l], [], []], [[m, l], [s], []], [[l], [s], [m]], [[s, l], [], [m]], [[l], [], [s, m]], [[], [l], [s, m]], [[s], [l], [m]], [[], [s, l], [m]], [[m], [s, l], []], [[s, m], [l], []], [[m], [l], [s]]]]

Move = m21

NextState = [[m], [l], [s]]

Move = m23

NextState = [[], [l], [m, s]]

Move = m31

NextState = [[s],[m,l],[]]

PathSoFar =

[[[s,m,l],[],[],[[m,l],[s,[]],[[l],[s,[m]]],[[s,l],[],[m]],[[l],[],[s,m]],[[s],[l],[m]],[],[s,l],[m]],[[m],[s,l],[],[[s,m,l],[],[]]]]

Move = m12

NextState = [[],[s,m,l],[]]

PathSoFar =

[[[s,m,l],[],[],[[m,l],[s,[]],[[l],[s,[m]]],[[s,l],[],[m]],[[l],[],[s,m]],[[s],[l],[m]],[],[s,l],[m]],[[m],[s,l],[],[[s,m,l],[],[]]]]

Move = m21

NextState = [[s],[m,l],[]]

Move = m23

NextState = [[],[m,l],[s]]

Move = m13

NextState = [[],[m,l],[s]]

Move = m21

NextState = [[m,s],[l],[]]

Move = m23

NextState = [[s],[l],[m]]

Move = m32

NextState = [[],[s,m,l],[]]

PathSoFar =

[[[s,m,l],[],[],[[m,l],[s,[]],[[l],[s,[m]]],[[s,l],[],[m]],[[l],[],[s,m]],[[s],[l],[m]],[],[s,l],[m]],[[m],[s,l],[],[[s,m,l],[],[]]]]

Move = m21

NextState = [[s],[m,l],[]]

PathSoFar =

[[[s,m,l],[],[],[[m,l],[s,[]],[[l],[s,[m]]],[[s,l],[],[m]],[[l],[],[s,m]],[[s],[l],[m]],[],[s,l],[m]],[[m],[s,l],[],[[s,m,l],[],[]]]]

Move = m12

NextState = [[],[s,m,l],[]]

Move = m13

NextState = [[],[m,l],[s]]

Move = m21

NextState = [[m,s],[l],[]]

Move = m23

```

NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[l],[m,s]]
Move = m21
NextState = [[l,m],[],[s]]
Move = m23
NextState = [[m],[],[l,s]]
Move = m31
NextState = [[s,m],[l],[]]
Move = m32
NextState = [[m],[s,l],[]]
Move = m21
NextState = [[l,s,m],[],[]]
Move = m23
NextState = [[s,m],[],[l]]
PathSoFar =
[[[s,m,l],[],[]], [[m,l],[s,[]]], [[l],[s],[m]], [[s,l],[],[m]], [[l],[],[s,m]], [[],[],[s,m]], [[s],[l],[m]], [[],[],[s,l],[m]], [[m],[s,l],[]], [[s,m],[l,[]]], [[s,m],[],[l]]]
Move = m12
NextState = [[m],[s],[l]]
PathSoFar =
[[[s,m,l],[],[]], [[m,l],[s,[]]], [[l],[s],[m]], [[s,l],[],[m]], [[l],[],[s,m]], [[],[],[s,m]], [[s],[l],[m]], [[],[],[s,l],[m]], [[m],[s,l],[]], [[s,m],[l,[]]], [[s,m],[],[l]], [[m],[s],[l]]]
Move = m12
NextState = [[],[m,s],[l]]
Move = m13
NextState = [[],[s],[m,l]]
PathSoFar =
[[[s,m,l],[],[]], [[m,l],[s,[]]], [[l],[s],[m]], [[s,l],[],[m]], [[l],[],[s,m]], [[],[],[s,m]], [[s],[l],[m]], [[],[],[s,l],[m]], [[m],[s,l],[]], [[s,m],[l,[]]], [[s,m],[],[l]], [[m],[s],[l]]]
Move = m21
NextState = [[s],[],[m,l]]

```

```

PathSoFar =
[[[s,m,l],[[],[]],[[m,l],[s,[]],[[],[l],[s],[m]],[[s,l],[[],[m]],[[],[],[s,m]]],[[],[],[s,m]],[[s],[[],[m]]],[[],[s,l],[m]],[[[m],[s,l],[]],[[s,m,l]]]]]

Move = m12

NextState = [[],[s],[m,l]]

Move = m13

NextState = [[],[[],[s,m,l]]]

PathSoFar =
[[[s,m,l],[[],[]],[[m,l],[s,[]],[[],[l],[s],[m]],[[s,l],[[],[m]],[[],[],[s,m]]],[[],[],[s,m]],[[s],[[],[m]]],[[],[s,l],[m]],[[[m],[s,l],[]],[[s,m,l]]]]]

SolutionSoFar = [m12,m13,m21,m13,m12,m31,m12,m31,m21,m23,m12,m13,m21,m13]

Solution ...

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

true

```

Task 8: Run the program to solve the 4 disk problem

```

?- solve.

PathSoFar = [[[s,m,l,h],[[],[]]]]

Move = m12

NextState = [[m,l,h],[s,[]]]

PathSoFar = [[[s,m,l,h],[[],[]],[[m,l,h],[s,[]]]]]

```

Move = m12

NextState = [[l,h],[m,s],[]]

Move = m13

NextState = [[l,h],[s],[m]]

PathSoFar = [[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]]]

Move = m12

NextState = [[h],[l,s],[m]]

Move = m13

NextState = [[h],[s],[l,m]]

Move = m21

NextState = [[s,l,h],[],[m]]

PathSoFar = [[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]]]

Move = m12

NextState = [[l,h],[s],[m]]

Move = m13

NextState = [[l,h],[],[s,m]]

PathSoFar = [[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]]]

Move = m12

NextState = [[h],[l],[s,m]]

PathSoFar = [[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]]]

Move = m12

NextState = [[],[h,l],[s,m]]

Move = m13

NextState = [[],[l],[h,s,m]]

Move = m21

NextState = [[l,h],[],[s,m]]

Move = m23

NextState = [[h],[],[l,s,m]]

Move = m31

NextState = [[s,h],[l],[m]]

PathSoFar = [[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]]]

Move = m12

NextState = [[h],[s,l],[m]]

```

PathSoFar =
[[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]]]

Move = m12

NextState = [[],[h,s,l],[m]]

Move = m13

NextState = [[],[s,l],[h,m]]

Move = m21

NextState = [[s,h],[l],[m]]

Move = m23

NextState = [[h],[l],[s,m]]

Move = m31

NextState = [[m,h],[s,l],[]]

PathSoFar =
[[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l],[]]]]

Move = m12

NextState = [[h],[m,s,l],[]]

Move = m13

NextState = [[h],[s,l],[m]]

Move = m21

NextState = [[s,m,h],[l],[]]

PathSoFar =
[[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l],[]],[[s,m,h],[l],[]]]]

Move = m12

NextState = [[m,h],[s,l],[]]

Move = m13

NextState = [[m,h],[l],[s]]

PathSoFar =
[[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l],[]],[[s,m,h],[l],[]],[[m,h],[l],[s]]]]]

Move = m12

NextState = [[h],[m,l],[s]]

PathSoFar =
[[[s,m,l,h],[],[]],[[m,l,h],[s],[]],[[l,h],[s],[m]],[[s,l,h],[],[m]],[[l,h],[],[s,m]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l],[]],[[s,m,h],[l],[]],[[m,h],[l],[s]]]]]

Move = m12

```

NextState = [[],[h,m,l],[s]]

Move = m13

NextState = [[],[m,l],[h,s]]

Move = m21

NextState = [[m,h],[l],[s]]

Move = m23

NextState = [[h],[l],[m,s]]

Move = m31

NextState = [[s,h],[m,l],[]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]]]],[[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]]

Move = m12

NextState = [[h],[s,m,l],[]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]]]],[[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]]

Move = m12

NextState = [[],[h,s,m,l],[]]

Move = m13

NextState = [[],[s,m,l],[h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]]]],[[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]]

Move = m21

NextState = [[s],[m,l],[h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]]]],[[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]]

Move = m12

NextState = [[],[s,m,l],[h]]

Move = m13

NextState = [[],[m,l],[s,h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]]]],[[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]]

Move = m21

NextState = [[m],[l],[s,h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]],[[s,h],[l],[m]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[[h],[s,m,l],[[]],[[[s],[m,l],[h]],[[[s],[m,l],[h]]],[[l],[s,m,l],[h]]],[[m,l],[s,h]],[[[m],[l],[s,h]]]

Move = m12

NextState = [[],[m,l],[s,h]]

Move = m13

NextState = [[],[l],[m,s,h]]

Move = m21

NextState = [[l,m],[[],[s,h]]

Move = m23

NextState = [[m],[[],[l,s,h]]

Move = m31

NextState = [[s,m],[l],[h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]],[[s,h],[l],[m]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[[h],[s,m,l],[[]],[[[s],[m,l],[h]],[[[s],[m,l],[h]]],[[m,l],[s,h]],[[[m],[l],[s,h]]],[[s,m],[l],[h]],[[m],[s,l],[h]]]

Move = m12

NextState = [[m],[s,l],[h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]],[[s,h],[l],[m]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[[h],[s,m,l],[[]],[[[s],[m,l],[h]],[[[s],[m,l],[h]]],[[m,l],[s,h]],[[[m],[l],[s,h]]],[[s,m],[l],[h]],[[m],[s,l],[h]]]

Move = m12

NextState = [[],[m,s,l],[h]]

Move = m13

NextState = [[],[s,l],[m,h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]],[[s,h],[l],[m]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[[h],[s,m,l],[[]],[[[s],[m,l],[h]],[[[s],[m,l],[h]]],[[m,l],[s,h]],[[[m],[l],[s,h]]],[[s,m],[l],[h]],[[m],[s,l],[h]]]

Move = m21

NextState = [[s],[l],[m,h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]],[[s,h],[l],[m]],[[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l],[s]],[[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[[h],[s,m,l],[[]],[[[s],[m,l],[h]],[[[s],[m,l],[h]]],[[m,l],[s,h]],[[[m],[l],[s,h]]],[[s,m],[l],[h]],[[m],[s,l],[h]]]

[m,h],[s,l],[],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l,[]],[[h],[s,m,l],[],[],[s,m,l],[h]],[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]]]

Move = m12

NextState = [[], [s, l], [m, h]]

Move = m13

NextState = [[], [l], [s, m, h]]

PathSoFar =

[[[s,m,l,h],[l,[]],[[m,l,h],[s,[]],[[l,h],[s],[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l,[]],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l,[]],[[h],[s,m,l],[],[],[s,m,l],[h]],[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]]],[[],[l],[s,m,h]]]

Move = m21

NextState = [[l],[],[s,m,h]]

PathSoFar =

[[[s,m,l,h],[l,[]],[[m,l,h],[s,[]],[[l,h],[s],[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l,[]],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l,[]],[[h],[s,m,l],[],[],[s,m,l],[h]],[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]]],[[],[l],[s,m,h]]]

Move = m12

NextState = [[], [l], [s, m, h]]

Move = m13

NextState = [[], [], [l, s, m, h]]

Move = m31

NextState = [[s,l],[],[m,h]]

PathSoFar =

[[[s,m,l,h],[l,[]],[[m,l,h],[s,[]],[[l,h],[s],[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l,[]],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l,[]],[[h],[s,m,l],[],[],[s,m,l],[h]],[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]]],[[],[l],[s,m,h]]]

Move = m12

NextState = [[l],[s],[m,h]]

PathSoFar =

[[[s,m,l,h],[l,[]],[[m,l,h],[s,[]],[[l,h],[s],[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]],[[s,h],[l],[m]],[[h],[s,l],[m]],[[m,h],[s,l,[]],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l,[]],[[h],[s,m,l],[],[],[s,m,l],[h]],[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]]],[[],[l],[s,m,h]]]

Move = m12

NextState = [[], [l, s], [m, h]]

Move = m13

NextState = [[], [s], [l, m, h]]

Move = m21

NextState = [[s,l],[[],[m,h]]]

Move = m23

NextState = [[l],[],[s,m,h]]

Move = m31

NextState = [[m,l],[s],[h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s],[[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[[m,h],[s,l],[[]]]],[[[s,m,h],[l],[[]],[[m,h],[l],[s]]],[[[h],[m,l],[s]],[[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]],[[[l],[s,m,l],[h]]],[[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[[m],[s,h]]],[[[s,m],[l],[h]]],[[[m],[s,l],[h]]],[[[l],[s,l],[m,h]]],[[[s],[l],[m,h]]],[[[l],[l],[s,m,h]]],[[[l],[l],[s,m,h]]],[[[s,l],[l],[[]]]],[[[m,h],[l],[s],[m,h]]],[[[m,l],[s],[h]]],[[[m,l],[s],[h]]]]]

Move = m12

NextState = [[l],[m,s],[h]]

Move = m13

NextState = [[l],[s],[m,h]]

Move = m21

NextState = [[s,m,l],[[],[h]]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s],[[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[[m,h],[s,l],[[]]]],[[[s,m,h],[l],[[]],[[m,h],[l],[s]]],[[[h],[m,l],[s]],[[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]],[[[l],[s,m,l],[h]]],[[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[[m],[s,h]]],[[[s,m],[l],[h]]],[[[m],[s,l],[h]]],[[[l],[s,l],[m,h]]],[[[s],[l],[m,h]]],[[[l],[l],[s,m,h]]],[[[l],[l],[s,m,h]]],[[[s,l],[l],[[]]]],[[[m,h],[l],[s],[m,h]]],[[[m,l],[s],[h]]],[[[m,l],[s],[h]]]]]

Move = m12

NextState = [[m,l],[s],[h]]

Move = m13

NextState = [[m,l],[[],[s,h]]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s],[[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[[m,h],[s,l],[[]]]],[[[s,m,h],[l],[[]],[[m,h],[l],[s]]],[[[h],[m,l],[s]],[[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]],[[[l],[s,m,l],[h]]],[[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[[m],[s,h]]],[[[s,m],[l],[h]]],[[[m],[s,l],[h]]],[[[l],[s,l],[m,h]]],[[[s],[l],[m,h]]],[[[l],[l],[s,m,h]]],[[[l],[l],[s,m,h]]],[[[s,l],[l],[[]]]],[[[m,h],[l],[s],[m,h]]],[[[m,l],[s],[h]]],[[[m,l],[s],[h]]]]]

Move = m12

NextState = [[l],[m],[s,h]]

PathSoFar =

[[[s,m,l,h],[[],[]],[[m,l,h],[s],[[]],[[l,h],[s],[m]],[[s,l,h],[[],[m]],[[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[[s,h],[l],[m]]],[[[h],[s,l],[m]]],[[[m,h],[s,l],[[]]]],[[[s,m,h],[l],[[]],[[m,h],[l],[s]]],[[[h],[m,l],[s]],[[[s,h],[m,l],[[]]]],[[[h],[s,m,l],[[]]]],[[[l],[s,m,l],[h]]],[[[s],[m,l],[h]]],[[],[m,l],[s,h]],[[[m],[s,h]]],[[[s,m],[l],[h]]],[[[m],[s,l],[h]]],[[[l],[s,l],[m,h]]],[[[s],[l],[m,h]]],[[[l],[l],[s,m,h]]],[[[l],[l],[s,m,h]]],[[[s,l],[l],[[]]]],[[[m,h],[l],[s],[m,h]]],[[[m,l],[s],[h]]],[[[m,l],[s],[h]]]]]

Move = m12

NextState = [[[],[l,m],[s,h]]]

Move = m13

NextState = [[], [m], [l, s, h]]

Move = m21

NextState = [[m, l], [], [s, h]]

Move = m23

NextState = [[l], [], [m, s, h]]

Move = m31

NextState = [[s, l], [m], [h]]

PathSoFar =

[[[s, m, l, h], [], []], [[[m, l, h], [s], []], [[l, h], [s], [m]], [[s, l, h], [], [m]], [[l, h], [], [s, m]], [[h], [l], [s, m]], [[s, h], [l], [m]], [[h], [s, l], [m]], [[m, h], [s, l], []], [[s, m, h], [l], []], [[m, h], [l], [s]], [[h], [m, l], [s]], [[s, h], [m, l], []], [[h], [s, m, l], []], [[s, m, l], [h]], [[s], [m, l], [h]], [[m, l], [s, h]], [[m], [l], [s, h]], [[s, m], [l], [h]], [[m], [s, l], [h]], [[l], [s, l], [m, h]], [[s], [l], [m, h]], [[l], [s, m, h]], [[l], [l], [s, m, h]], [[s, l], [m], [h]], [[s, l], [m], [h]]]]

Move = m12

NextState = [[l], [s, m], [h]]

PathSoFar =

[[[s, m, l, h], [], []], [[[m, l, h], [s], []], [[l, h], [s], [m]], [[s, l, h], [], [m]], [[l, h], [], [s, m]], [[h], [l], [s, m]], [[s, h], [l], [m]], [[h], [s, l], [m]], [[m, h], [s, l], []], [[s, m, h], [l], []], [[m, h], [l], [s]], [[h], [m, l], [s]], [[s, h], [m, l], []], [[h], [s, m, l], []], [[s, m, l], [h]], [[s], [m, l], [h]], [[m, l], [s, h]], [[m], [l], [s, h]], [[s, m], [l], [h]], [[m], [s, l], [h]], [[l], [s, l], [m, h]], [[s], [l], [m, h]], [[l], [s, m, h]], [[l], [l], [s, m, h]], [[s, l], [m], [h]], [[s, l], [m], [h]]]]

Move = m12

NextState = [[], [l, s, m], [h]]

Move = m13

NextState = [[], [s, m], [l, h]]

PathSoFar =

[[[s, m, l, h], [], []], [[[m, l, h], [s], []], [[l, h], [s], [m]], [[s, l, h], [], [m]], [[l, h], [], [s, m]], [[h], [l], [s, m]], [[s, h], [l], [m]], [[h], [s, l], [m]], [[m, h], [s, l], []], [[s, m, h], [l], []], [[m, h], [l], [s]], [[h], [m, l], [s]], [[s, h], [m, l], []], [[h], [s, m, l], []], [[s, m, l], [h]], [[s], [m, l], [h]], [[m, l], [s, h]], [[m], [l], [s, h]], [[s, m], [l], [h]], [[m], [s, l], [h]], [[l], [s, l], [m, h]], [[s], [l], [m, h]], [[l], [s, m, h]], [[l], [l], [s, m, h]], [[s, l], [m], [h]], [[s, l], [m], [h]]]]

Move = m21

NextState = [[s], [m], [l, h]]

PathSoFar =

[[[s, m, l, h], [], []], [[[m, l, h], [s], []], [[l, h], [s], [m]], [[s, l, h], [], [m]], [[l, h], [], [s, m]], [[h], [l], [s, m]], [[s, h], [l], [m]], [[h], [s, l], [m]], [[m, h], [s, l], []], [[s, m, h], [l], []], [[m, h], [l], [s]], [[h], [m, l], [s]], [[s, h], [m, l], []], [[h], [s, m, l], []], [[s, m, l], [h]], [[s], [m, l], [h]], [[m, l], [s, h]], [[m], [l], [s, h]], [[s, m], [l], [h]], [[m], [s, l], [h]], [[l], [s, l], [m, h]], [[s], [l], [m, h]], [[l], [s, m, h]], [[l], [l], [s, m, h]], [[s, l], [m], [h]], [[s, l], [m], [h]]]]

Move = m12

NextState = [[], [s, m], [l, h]]

Move = m13

NextState = [[], [m], [s,l,h]]

PathSoFar =

```

[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]]],[[s,h],[l],[m]]],[[h],[s,l],[m]]],[[m,h],[s,l],[[]]],[[s,m,h],[l,[]],[[m,h],[l],[s]]],[[h],[m,l],[s]],[[s,h],[m,l],[[]]],[[h],[s,m,l],[[]]],[[],[s,m,l],[h]],[[s],[m,l],[h]],[[],[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[],[s,l],[m,h]],[[s],[l],[m,h]],[[],[l],[s,m,h]],[[l],[l],[s,m,h]],[[s],[l],[[],[m,h]]],[[l],[s],[m,h]],[[],[l],[s,m],[h]],[[],[s,m],[l,h]],[[s],[m],[l,h]],[[],[m],[s,l,h]]]

```

Move = m21

NextState = [[m],[],[s,l,h]]

PathSoFar =

```
[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[[h],[l],[s,m]]],[[s,h],[[],[m]]],[[[h],[s,l],[m]]],[[m,h],[s,l,[]]],[[s,m,h],[[],[]],[[m,h],[[],[s]]],[[[h],[m,l],[s]]],[[s,h],[[],[m,l]]],[[[h],[s,m,l],[[]]]],[[],[s,m,l],[h]],[[s],[m,l],[h]],[[],[m,l],[s,h]],[[[m],[l],[s,h]]],[[s,m],[[],[h]]],[[[m],[s,l],[h]]],[[],[s,l],[m,h]],[[s],[[],[m,h]]],[[],[l],[s,m,h]],[[],[l],[s,m,h]],[[s,l],[[],[]],[[m,h],[[],[s]]],[[[l],[s],[h]]],[[l],[m],[s,h]],[[s,l],[[],[m],[h]]],[[],[l],[s,m],[h]],[[],[s,m],[l,h]],[[s],[m],[l,h]],[[],[m],[s,l,h]],[[m],[[],[s,l,h]]]]]
```

Move = m12

NextState = [[], [m], [s, l, h]]

Move = m13

NextState = [[],[],[m,s,l,h]]

Move = m31

NextState = [[s,m],[],[l,h]]

PathSoFar =

```
[[[s,m,l,h],[[],[]],[[m,l,h],[s,[]],[[l,h],[s,[m]]],[[s,l,h],[[],[m]]],[[l,h],[[],[s,m]]],[[h],[[],[s,m]]],[[s,h],[[],[m]]],[[h],[[],[s,l],[m]]],[[m,h],[[],[s,l]]],[[s,m,h],[[],[l]]],[[m,h],[[],[l,s]]],[[h],[[],[m,l],[s]]],[[s,h],[[],[m,l]]],[[h],[[],[s,m,l]]],[[],[l],[s,m,l],[h]],[[s],[[],[m,l],[h]]],[[],[l],[m,l],[s,h]],[[m],[[],[l],[s,h]]],[[s,m],[[],[l],[h]]],[[m],[[],[s,l],[h]]],[[],[l],[s,l],[m,h]],[[s],[[],[m,h]]],[[],[l],[s,m,h]],[[],[l],[s,m,h]],[[s,l],[[],[l]]],[[m,h],[[],[s],[m,h]]],[[m,l],[[],[s],[h]]],[[s,m,l],[[],[h]]],[[m,l],[[],[s,h]]],[[],[l],[m],[s,h]],[[s,l],[[],[m],[h]]],[[],[l],[s,m],[h]],[[],[l],[s,m],[l,h]],[[s],[[],[m],[l,h]]],[[],[m],[[],[s,l,h]]],[[s,m],[[],[l,h]]]]
```

Move = m12

NextState = [[m],[s],[l,h]]

PathSoFar =

Move = m12

NextState = [[], [m,s], [l,h]]

Move = m13

NextState = [[], [s], [m, l, h]]

Move = m21

NextState = [[s],[],[m,l,h]]

PathSoFar =

```
[[[s,m,l,h],[l,[]],[[m,l,h],[s,[]],[[l,h],[s,[m]],[[s,l,h],[[],[m]],[[l,h],[[],[s,m]]],[[h],[l],[s,m]]],[[s,h],[l],[m]]],[[h],[s,l],[m]]],[[m,h],[s,l],[[]],[[s,m,h],[l,[]],[[m,h],[l,[]],[[s,h],[l,[]],[[h],[m,l],[s]],[[s,h],[m,l],[[]],[[h],[s,m,l],[[]],[[l],[s,m,l],[h]],[[s],[m,l],[h]],[],[[m,l],[s,h]],[[m],[l],[s,h]],[[s,m],[l],[h]],[[m],[s,l],[h]],[[l],[s,l],[m,h]],[[s],[l],[m,h]],[[l],[l],[s,m,h]],[[l],[s,m,h]],[[s,l],[l,[]],[[m,h],[l,[]],[[l],[s,m,h]],[[m,l],[s,h]],[[l],[m],[s,h]],[[s,l],[m],[h]],[[l],[s,m],[h]],[[l],[s,m],[l,h]],[[s],[m],[l,h]]]]]
```

Move = m12

NextState = [[], [s], [m,l,h]]

Move = m13

NextState = [[],[],[s,m,l,h]]

PathSoFar =

SolutionSoFar =

[m12,m13,m21,m13,m12,m31,m12,m31,m21,m13,m12,m31,m12,m13,m21,m13,m21,m31,m12,m13,m21,m13,m21,m13,m21,m13,m21,m13,m21,m13,m21,m13,m21,m13]

Solution ...

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 3 to tower 1.

Move the top Disk from tower 1 to tower 2.

Move the top Disk from tower 1 to tower 3.

Move the top Disk from tower 2 to tower 1.

Move the top Disk from tower 1 to tower 3.

True

Task 9: Review your code and archive it

```

% -----
% -----
% --- File: towers_of_hanoi.pro
% --- Line: Program to solve the Towers of Hanoi problem
% -----
:- consult('inspector.pro').
% -----
% --- make_move(S,T,SSO) :: Make a move from state S to state T by SSO

```

```

make_move(TowersBeforeMove,TowersAfterMove,m12) :-
m12(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m13) :-
m13(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m21) :-
m21(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m23) :-
m23(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m31) :-
m31(TowersBeforeMove,TowersAfterMove).
make_move(TowersBeforeMove,TowersAfterMove,m32) :-
m32(TowersBeforeMove,TowersAfterMove).

```

```

m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower2Before = L,
Tower2After = [H|L].
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower3Before = L,
Tower3After = [H|L].
m21([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower2Before = [H|T],
Tower2After = T,
Tower1Before = L,
Tower1After = [H|L].

```

```

m23([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
Tower2Before = [H|T],
Tower2After = T,
Tower3Before = L,
Tower3After = [H|L].
m31([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
Tower3Before = [H|T],
Tower3After = T,
Tower1Before = L,
Tower1After = [H|L].
m32([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
Tower3Before = [H|T],
Tower3After = T,
Tower2Before = L,
Tower2After = [H|L].

```

% -----

% --- valid_state(S) :: S is a valid state

```

valid_state([Tower1,Tower2,Tower3]) :-
valid_tower(Tower1),
valid_tower(Tower2),
valid_tower(Tower3).

```

```
valid_tower([]).
```

```

valid_tower([t]).
valid_tower([s]).
valid_tower([m]).
valid_tower([l]).
valid_tower([h]).
```

```

valid_tower([t, s]).
valid_tower([t, m]).
valid_tower([t, l]).
valid_tower([t, h]).
```

```

valid_tower([s, m]).
valid_tower([s, l]).
valid_tower([s, h]).
```

```

valid_tower([m, h]).  

valid_tower([m, l]).  
  

valid_tower([l, h]).  
  

valid_tower([t, s, h]).  

valid_tower([t, s, l]).  

valid_tower([t, s, m]).  

valid_tower([t, m, h]).  

valid_tower([t, m, l]).  
  

valid_tower([s, m, l]).  

valid_tower([s, m, h]).  

valid_tower([s, l, h]).  
  

valid_tower([m, l, h]).  
  

valid_tower([t, s, l, h]).  

valid_tower([t, m, l, h]).  

valid_tower([t, s, m, h]).  

valid_tower([t, s, m, l]).  
  

valid_tower([s, m, l, h]).  
  

valid_tower([t, s, m, l, h]).  
  

% -----  

% --- solve(Start,Solution) :: succeeds if Solution represents a path  

% --- from the start state to the goal state.  

solve :-  

    extend_path([[s,m,l],[],[]],[],Solution),  

    write_solution(Solution).  

extend_path(PathSoFar,SolutionSoFar,Solution) :-  

    PathSoFar = [[[],[],[s,m,l]]|_],  

    showr('PathSoFar',PathSoFar),  

    showr('SolutionSoFar',SolutionSoFar),  

    Solution = SolutionSoFar.  

extend_path(PathSoFar,SolutionSoFar,Solution) :-  


```

```

PathSoFar = [CurrentState|_],
showr('PathSoFar',PathSoFar),
make_move(CurrentState,NextState,Move),
show('Move',Move),
show('NextState',NextState),
not(member(NextState,PathSoFar)),
valid_state(NextState),
Path = [NextState|PathSoFar],
Soln = [Move|SolutionSoFar],
extend_path(Path,Soln,Solution).

% -----
% --- write_sequence_reversed(S) :: Write the sequence, given by S,
% --- expanding the tokens into meaningful strings.
write_solution(S) :-
nl, write('Solution ...'), nl, nl, reverse(S,R), write_sequence(R),nl.

write_sequence([]).

write_sequence([H | T]) :-
elaborate(H, E), write(E), nl, write_sequence(T).

elaborate(m12, E) :-
E = "Move the top Disk from tower 1 to tower 2.".
elaborate(m13, E) :-
E = "Move the top Disk from tower 1 to tower 3.".
elaborate(m21, E) :-
E = "Move the top Disk from tower 2 to tower 1.".
elaborate(m23, E) :-
E = "Move the top Disk from tower 2 to tower 3.".
elaborate(m31, E) :-
E = "Move the top Disk from tower 3 to tower 1.".
elaborate(m32, E) :-
E = "Move the top Disk from tower 3 to tower 2.".

% -----
% --- Unit test programs

```

```
test__m12 :-
```

```
write('Testing: move_m12\n'), TowersBefore = [[t,s,m,l,h],[],[]], trace(",'TowersBefore',TowersBefore),  
m12(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__m13 :-
```

```
write('Testing: move_m13\n'), TowersBefore = [[t,s,m,l,h],[],[]], trace(",'TowersBefore',TowersBefore),  
m13(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__m21 :-
```

```
write('Testing: move_m21\n'), TowersBefore = [[], [t,s,m,l,h], []], trace(",'TowersBefore',TowersBefore),  
m21(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__m23 :-
```

```
write('Testing: move_m23\n'), TowersBefore = [[], [t,s,m,l,h], []], trace(",'TowersBefore',TowersBefore),  
m23(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__m31 :-
```

```
write('Testing: move_m31\n'), TowersBefore = [[], [], [t,s,m,l,h]], trace(",'TowersBefore',TowersBefore),  
m31(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__m32 :-
```

```
write('Testing: move_m32\n'), TowersBefore = [[], [], [t,s,m,l,h]], trace(",'TowersBefore',TowersBefore),  
m32(TowersBefore,TowersAfter), trace(",'TowersAfter',TowersAfter).
```

```
test__valid_state :-
```

```
write('Testing: valid_state\n'),
```

```
test__vs([[l,t,s,m,h],[],[]]),
```

```
test__vs([[t,s,m,l,h],[],[]]),
```

```
test__vs([[[],[h,t,s,m],[l]]]),
```

```
test__vs([[[],[t,s,m,h],[l]]]),
```

```
test__vs([[[],[h],[l,m,s,t]]]),
```

```
test__vs([[[],[h],[t,s,m,l]])].
```

```
test__vs(S) :-
```

```
valid_state(S),
```

```
write(S),
```

```
write(' is valid.'),
```

```
nl.
```

```
test__vs(S) :-
```

```
write(S),
```

```
write(' is invalid.'),
```

```
nl.
```

```
test__write_sequence :-
```

```
write('First test of write_sequence ...'),
```

```
nl,  
write_sequence([m31,m12,m13,m21]),  
write('Second test of write_sequence ...'),  
nl,  
write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```