Elliptical PDE Problem : Laplace Equation

Determine the steady-state 2-dimensional heat distribution in a thin metal plate using Central Differences Method and Jacobi iteration.

Inner region:

$$\frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) = 0$$

$0 < x < 0.5, 0 < y < 0.5$

Boundary conditions:

Two adjacent boundaries are held at a constant 0° Celsius. The heat on the other two adjacent boundaries is also held constant, and increases linearly from 0° to a maximum of 100° (where the two edges meet).

Grid:

$n = m = 4$.

Run program with different tolerances. Approximate the solution to a minimum tolerance of 1×10^{-10}.

For each tolerance used: output the tolerance value, the number of iterations to achieve convergence, and the resulting temperatures.

Make your program general enough so that it will still work when the value of n (m) is changed.

Hint: use DecimalFormat for "pretty printing" the resulting temperatures.

Extra credit:

1. Make it a Swing application, displaying the temperatures for each iteration.
2. Provide a GUI interface where the user can specify, for each simulation, the following parameters:
   - The constant (low) temperature on the two adjacent boundaries
   - the low and high heat temperatures on the other two adjacent boundaries
   - the length of x ($=y$)
   - the grid size n ($=m$).