1. Show that for every subgroup $H$ of $S_n$ for $n \geq 2$, either all the permutations in $H$ are even or exactly half of them are even.

2. Consider $S_n$ for a fixed $n \geq 2$ and let $\sigma$ be a fixed odd permutation. Show that every odd permutation in $S_n$ is a product of $\sigma$ and some permutation in $A_n$.

3. Let $G$ be a group and let $a$ be a fixed element of $G$. The map $\lambda_a : G \rightarrow G$, given by $\lambda_a(g) = ag$ for $g \in G$, is a permutation of the set $G$. Show that $H = \{\lambda_a | a \in G\}$ is a subgroup of $S_G$, the group of all permutations of $G$. 