Day 5: more on §2 pg 25: 17 - 23, 29, redo prev. on " 32, 36, 37

As Friday thru today includes x'd defn. Start studying §4 pg 45: 1 - 19 odd

Back to day 1

Defn: We say a motion \( \alpha \) followed by \( \beta \) denoted \( \beta \alpha \) is equal to the motion \( \gamma \) if upon performing the motion \( \alpha \) and then the motion \( \beta \), the result is the same as performing \( \gamma \).

Recall \( S = \{ \rho_0, \rho_1, \rho_2, \mu_1, \mu_2, \mu_3 \} \)

ex: \( \mu_1 \rho_1 = (1\ 2\ 3)(1\ 2\ 3) = (1\ 2\ 3) = \mu_a \)

\[ \rho_1 \text{ followed by } \mu_1 \]

Note: When we are making the table,

1) We needed no new notation (follow by on \( S \) is closed). That is " followed by " is a binary operation on \( S \).

2) Combining with \( \rho_0 \) (" do nothing ") on either side results in the same motion ( \( \rho_0 \) is the identity).

3) For each motion there is a motion that " undoes " the motion (every motion has an inverse).
4) Followed by is associative. That is,
   \[ d(\beta \gamma) = (d\beta)\gamma \]
   (this follows from composition of functions is associative)

Is followed by commutative?

\[ \mu_1 = (1\ 2\ 3)(1\ 3\ 2) = (1\ 2\ 3) \neq \mu_1 \mu_1 \]

* Defn: Let \( G \) be a set along with a binary operation \( * \) on \( G \) which satisfies
  \[ g_1: \text{The binary operation } * \text{ is associative on } G. \]

  \[ g_2: \text{There is an element } e \text{ in } G \]
  \[ \text{such that } e*a = a = a*e \]
  \[ \forall a \in G \]

  \[ g_3: \text{For each } a \in G \text{ there is an element } b \in G \]
  \[ \text{such that } a*b = e = b*a \]

Then \( G \) is called a group.

Ex (main) \( S_3 \) is a group (here \( S_3 \) is our \( S \))
Remark: The formal notation for a group \( < G, \ast > \)

e.g.: \( < \mathbb{Z}, + > \) is a group.
\( < \mathbb{Z}, \cdot > \) \( \text{no} \ y \neq 3 \)
\( < \mathbb{R}, \cdot > \)