DAY 34:

Think:

Let $H$ be a subgroup of a group $G$.

The partition of $G$ into left cosets of $H$ is the same as the partition into right cosets of $H$.

\[ \forall g \in G, \quad gH = Hg \]

If II is not automatic, I implies for $g \in G \exists j \in G \exists gH = Hj$.

For Wed one redo.

At Fri on something THW #7 like, and computations from Sec. 11: 1-20; 21-25; defn isomorphisms and/or Lagrange.

Last time: $\mathbb{Z}_2 \times \mathbb{Z}_3$ was isomorphic to $\mathbb{Z}_6$.

$\mathbb{Z}_2 \times \mathbb{Z}_4$ was not iso. to $\mathbb{Z}_8$. 
Theorem: The group \( \mathbb{Z}_m \times \mathbb{Z}_n \) is isomorphic to \( \mathbb{Z}_{mn} \) iff \( m \) and \( n \) are relatively prime.

**Proof:** outline

(\( \Rightarrow \)) by showing the contrapositive. That is, if \( m, n \) not relatively prime \( \Rightarrow \) \( \mathbb{Z}_m \times \mathbb{Z}_n \) is not iso. to \( \mathbb{Z}_{mn} \)

(\( \Leftarrow \)) if \( m, n \) are rel. prime then \((1, 1)\) generates \( \mathbb{Z}_m \times \mathbb{Z}_n \), shows iso.

Cor: The group \( \prod_{i=1}^{n} \mathbb{Z}_{m_i} \) is cyclic, hence iso. to \( \mathbb{Z}_{m_1 \cdot m_2 \cdots m_n} \) iff the \( m_i \)'s are pairwise relatively prime.

Ex: \( \mathbb{Z}_2 \times \mathbb{Z}_{15} \times \mathbb{Z}_7 \) is iso to \( \mathbb{Z}_{210} \)

\( \mathbb{Z}_2 \times \mathbb{Z}_{10} \times \mathbb{Z}_3 \) is not iso to \( \mathbb{Z}_{60} \)

What is the order of \((1, 1, 1)\) in \( \mathbb{Z}_2 \times \mathbb{Z}_{10} \times \mathbb{Z}_3 \)

\( 2 \cdot (1, 1, 1) = (1, 1, 1) + (1, 1, 1) = (0, 2, 2) \)

etc. \( \text{lcm} (2, 10, 3) = 30 \)

Notation: OLD (think algebra) \( 4 \times = x + x + x + x \)

\( x + x = 2x \)

In absence of notation

Same here: \( 3 \cdot (1, 1, 1) \) in \( \mathbb{Z}_2 \times \mathbb{Z}_{10} \times \mathbb{Z}_3 \)

\( = (1, 3, 0) \)
Theorem: Let \((a_1, a_2, \ldots, a_n) \in \prod_{i=1}^{n} G_i\). If \(a_i\) is of finite order \(r_i\) in \(G_i\), then the order of \((a_1, a_2, \ldots, a_n)\) in \(\prod_{i=1}^{n} G_i\) is \(\text{lcm}\{r_1, r_2, \ldots, r_m\}\).
24. \[ 720 = 2^4 \times 3^2 \times 5 \]

\[ \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \]
\[ \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \]
\[ \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \]
\[ \mathbb{Z}_{10} \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \]
\[ \mathbb{Z}_8 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \]

Etc.

Subgroups of \[ \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4 \] isom. to \[ \mathbb{V} \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \]

Make a \( \mathbb{V} \) (when \( G \) is abelian)

So, \[ \{ (0,0,0), (1,0,0), (0,1,0), (1,1,0) \} \]
\[ \{ (0,0,0), (0,0,2), (0,1,2), (0,1,0) \} \]
\[ \{ (0,0,0), (0,0,2), (1,0,0), (1,0,2) \} \]

more...

\[ \{ (0,0,0), (1,0,2), (1,0,1) \} \]

\[ \{ (0,0,0), (0,0,1), (0,0,2), (0,0,3) \} \cong \mathbb{Z}_4 \]

So NO

\[ \{ (0,0,0), (1,1,0), (0,0,2), (1,1,2) \} \]