DAY 3

THW: pg 8: 14b, 31, 32

(1) You must state the problem
(2) you have to write complete well-written math. sentences - word processed.

ex: see DAY 2

Study section 2 pg 25: 1-13

Defn: A binary operation on a set S is a function mapping $S \times S$ into S.

Notation: For each $(a,b) \in S \times S$ denote $\ast ((a, b)) = a \ast b$.

ex: $+$ on $\mathbb{R}$ is a binary operation

$+$ on $\mathbb{Z}$

Defns: Let $\ast$ be a binary operation on a set S and let $H$ be a subset of S. The subset $H$ is closed under $\ast$ if for all $a, b \in H$, $a \ast b \in H$.

In this case we say restricting $\ast$ to $H$ gives the induced operation of $\ast$ on $H$. 
ex (see pg 22 & 27)

Let $F = \{ f \mid f: \mathbb{R} \to \mathbb{R} \}$.

Let $f, g \in F$ we define $f + g$

to be $(f + g)(x) = f(x) + g(x)$

function $\leftarrow$ some real number

$\uparrow$ of functions addition

Let $f, g \in F$ we define

$(f \circ g)(x) = f(g(x))$

LHS is defined by RHS

Defs: (1) A binary operation $\ast$ on a set $S$ is commutative if $\forall a, b \in S$,

$a \ast b = b \ast a$.

(2) A binary operation $\ast$ on a set $S$

is associative if $\forall a, b, c \in S$,

$(a \ast b) \ast c = a \ast (b \ast c)$.

Note: pay particular attention to pg 23

ex 2.13
Remark: binary operations can be given by a table.

Ex: Let $S = \{a, b, c\}$. The following table defines a binary operation on $S$.

$$
\begin{array}{c|ccc}
& a & b & c \\
\hline
a & a & c & b \\
b & c & b & a \\
c & b & b & c \\
\end{array}
$$

Back to Day 1 we had "motions". The notion of followed by is a binary operation on the set of motions.
Show $|\{a, b\}| = |\{c, d\}|$

where $a, b, c, d \in \mathbb{R}$

Think: what does same card mean?

A function from one to the other that is 1-1 and onto.

A side ex: $\mathbb{Z} = \{\ldots, -2, 0, 2, 4, \ldots \}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots \}$

Here we'd define $f: \mathbb{Z} \to 2\mathbb{Z}$ by $f(n) = 2n$. Now 1-1 onto

Note $2\mathbb{Z} \subseteq \mathbb{Z}$

TH W: P1 say what you want to do

P2 Consider $f$: here to there defined

P3 Show $f$ is 1-1

P4 Show $f$ is onto

P5 Wrap