DAY 26: Keep working in §9

Fri: on all of §8
§9: 1-13 all
plus 2 *'s.

Fri THWS is due

ex: Find the orbits of \( \sigma = (1 \ 2 \ 3 \ 4 \ 5 \ 6) \in S_6 \)

\[ [1] = \{1, 5, 4, 6, 2\} \]
\[ [3] = \{3\} \]

cycle notation: \( (1 \ 5 \ 4 \ 6 \ 2) \)

Defn: A permutation \( \sigma \) of \( S_n \) is a cycle if it has at most one orbit containing more than one element. The length of a cycle is the number of elements in its largest orbit.

ex: a) \( \sigma \) from last time; \( \sigma = (1 \ 6 \ 2 \ 4 \ 5)(3 \ 7)(8 \ 10) \) is not a cycle.

b) \( \sigma = (1 \ 5 \ 4 \ 6 \ 2) \) from above is a cycle of length five,
Remark: Alternately, an expression of the form \((a_1, a_2, a_3, \ldots, a_n)\) is a cycle of length \(n\). (Here \(n = 7\)).

**Example:** Consider \((2, 5, 6, 3)\) in \(S_7\). This is a cycle of length 4 corresponding to the array notation \((1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)\).

**Theorem:** Every permutation \(\sigma\) of a finite set can be written as a product of disjoint cycles.

Consequence of the definition of orbits which is based on eq. classes.

**Example:**
\[ d = (1 \ 3 \ 4 \ 6), \quad \beta = (3 \ 4 \ 5) \quad \text{in} \quad S_6 \]

Find \(d\beta = (1 \ 3 \ 4 \ 6)(3 \ 4 \ 5)\)
\[ = (1 \ 3 \ 6)(4 \ 5) \]

b) first \(\sigma\) (other day):
\[ \sigma = (1 \ 6 \ 2 \ 4 \ 5)(3 \ 7)(8 \ 10) \]
\[ = (8 \ 10)(3 \ 7)(1 \ 6 \ 2 \ 4 \ 5) \]
Remark: M ult. of disjoint cycles is commutative.

ex:  Find \((14)(13)(12) = (1\ 2\ 3\ 4)\)
ex:  \((1\ 2\ 3) = (13)(12)\)

Defn: A cycle of length \(2\) is called a transposition.

ex:  Write \((15\ 3\ 2)\) as a product of transpositions.

\((15\ 3\ 2) = (12)(13)(15)\)  quick check.
\((15\ 3\ 2) = (12)(13)(14)(14)(15)\)

Remark: Any permutation of a finite set of at least \(2\) elements can be written a product of transpositions.

Theorem: If a permutation \(\sigma\) can be expressed as the product an even (odd) number of transpositions then every decomposition of \(\sigma\) must have an even (odd) number of transpositions.

Defn: A permutation that can be expressed as a product of an even number of transpositions is called an even permutation.  (Similarly odd permutation)