Test Review

Dfn: Let \( A \) be a nonempty set. A permutation of \( A \) is a function \( \sigma : A \rightarrow A \) that is both one-to-one and onto \( A \).

Examples

\[ A = \{1, 2, 3\} \]
\[ \sigma : A \rightarrow A \text{ where } \sigma(1) = 2, \sigma(2) = 3 \]
\[ \text{and } \sigma(3) = 1. \]

Clearly \( \sigma \) is both one-to-one and onto, so \( \sigma \) is a permutation of \( A \).

Notation

This permutation \( \sigma \) can be written as \( \sigma = (1 \ 2 \ 3) \).

Let \( \beta = (3 \ 2 \ 1) \)

How many permutations of \( A = \{1, 2, 3\} \) are there?

There are \( 3 \cdot 2 \cdot 1 = 6 = 3! \) permutations of \( A = \{1, 2, 3\} \).

In general there will be \( n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 = n! \) permutations of the set \( A = \{1, 2, 3, \ldots, n\} \).

The set of all \( n! \) permutations of \( A = \{1, 2, 3, \ldots, n\} \)
is denoted by \( S_n \).
We will make a group out of $S_n$, called the symmetric group on $n$ symbols.

We need a binary operation for $S_n$. If $\sigma: A \rightarrow A$ and $\beta: A \rightarrow A$ then $(\sigma \circ \beta): A \rightarrow A$ whose for all $a \in A$

$(\sigma \circ \beta)(a) = \sigma(\beta(a))$.

- Function composition is always associative so our binary operation $\circ$ is associative.