DAY 2: Reminder - There will be HW assigned Friday which will be collected Mon.

Redo: Study §0; more pg 8: 12, 14, 29-35 (skipping §1)

From section 0 - Sets and Relations

Set is a primitive concept
   elements, empty set, notation
   well-defined

subset $A \subseteq B$

$\uparrow$

set $A$ contained in $B$

or equal to $B$

we use $A \subseteq B$ in book $A \subseteq B$

Relations between sets
functions as relations, mapping,
domain, codomain, range

Cardinality
same cardinality iff $\exists$ a 1-1, onto
function from one to the other, inverse functions, infinite sets (some new stuff)

Partitions and Equivalence relations
defn of eq. relation (reflexive,
symmetric and transitive)
connection between eq. relations
   and partitions.
For any set $A$, finite or infinite, let $B^A$ be the set of all functions mapping $A$ into the set $B = \{0, 1\}$. Show the cardinality of $B^A$ is the same as the cardinality of the set $\mathcal{P}(A)$.

Thinking: We want to show $|B^A| = |\mathcal{P}(A)|$.

Note alternate ugly notation for $\mathcal{P}(A)$ is $2^A$.

Recall $\mathcal{P}(A)$ is the set of all subsets of $A$:

$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Now if $f \in B^A$ then $f: A \to B$ where $B = \{0, 1\}$.

We want a map $G: B^A \to \mathcal{P}(A)$ that is 1-1 and onto to show $|B^A| = |\mathcal{P}(A)|$.

Consider $G: B^A \to \mathcal{P}(A)$ (an element in here is a subset of $A$) and onto to show $|B^A| = |\mathcal{P}(A)|$.

$$G(f) = \{x \in A \mid f(x) = 1\}$$

Now we must show $G$ is 1-1 and onto.
Example of a proof that a given relation is an equivalence relation.

Ex: Define a relation $\sim$ on $\mathbb{Z}$ by $n \sim m$ if $n - m$ is divisible by 3. Then $\sim$ is an equivalence relation on $\mathbb{Z}$.

proof:

We want to show $\sim$ is an equivalence relation on $\mathbb{Z}$. We do so by showing $\sim$ satisfies the reflexive, symmetric and transitive properties.

Let $n \in \mathbb{Z}$. Then $n - n = 0 = 3(0)$ Hence $n - n$ is divisible by 3. We have shown $\forall n \in \mathbb{Z}, \ n \sim n$ which shows $\sim$ satisfies the reflexive property.

Let $n, m \in \mathbb{Z}$ such that $n \sim m$. Then, $n - m$ is divisible by 3. Thus $\exists k \in \mathbb{Z}$ such that $n - m = 3k$, which implies $m - n = 3(-k)$ where $-k$ is an integer. This shows $m - n$ is divisible by 3. So $m \sim n$. We have shown $\forall n, m \in \mathbb{Z}, \ n \sim m \Rightarrow m \sim n$. Hence $\sim$ satisfies the symmetric property.

Let $n, m, p \in \mathbb{Z}$ such that $n \sim m$ and $m \sim p$. Then $n - m$ and $m - p$ are divisible by 3. Thus $\exists k, j \in \mathbb{Z}$ such that $n - m = 3k$ and $m - p = 3j$. We see:

$$n - p = (n - m) + (m - p) = 3k + 3j = 3(k + j).$$

That is, $n - p = 3(k + j)$ where $k + j$ is an integer. This shows $n - p$ is divisible by 3. So $n \sim p$. We have shown $\forall n, m, p \in \mathbb{Z}, \ n \sim m$ and $m \sim p$ together imply $n \sim p$. Hence $\sim$ satisfies the transitive property.

We have shown $\sim$ satisfies the reflexive, symmetric and transitive properties hence $\sim$ is an equivalence relation on $\mathbb{Z}$. 