DAY II: Q3 on Friday on §3 and §4 1-19

our *'d defn of isomorphism of binary structures

There will be a question concerning matrices

Recall: Let $A \in M_2(\mathbb{R})$ then $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ for some $a, b, c, d \in \mathbb{R}$. We know $\det(A) = ad - bc$ (LHS is being defined by RHS)

Further $\det(AB) = (\det A) \cdot (\det B)$

mult of matrices

or whatever mult of the end

Remark: Isomorphisms preserve structural properties:

ex: Show the property that * is commutative structural property of $\langle S, * \rangle$.

proof:

Suppose $S$ is a binary structure with binary operation * being commutative. We want to show if $\Phi: S \rightarrow T$ is an isomorphism of $\langle S, * \rangle$ with $\langle T, @ \rangle$ then @ is commutative.

Let $\Phi: S \rightarrow T$ be an isomorphism of $\langle S, * \rangle$ with a binary structure $\langle T, @ \rangle$. We want to show @ is commutative. That is, we want to show $\forall t_1, t_2 \in T$ that $t_1 \circ t_2 = t_2 \circ t_1$.

Let $t_1, t_2 \in T$. Since $\Phi: S \rightarrow T$ is one-to-one and onto then $\Phi$ has an inverse. So $\exists s_1, s_2 \in S$ such that $\Phi(s_1) = t_1$ and $\Phi(s_2) = t_2$. Thus $t_1 \circ t_2 = \Phi(s_1) \circ \Phi(s_2)$ (substitution property of homomorphism preservation of $*$ is commutative) $= \Phi(s_1 \ast s_2)$ (since $\Phi(s_1) \circ \Phi(s_2) = \Phi(s_1 \ast s_2)$)

$= t_2 \circ t_1$ (since $\Phi(s_2) = t_2$ and $\Phi(s_1) = t_1$). That is, $t_1 \circ t_2 = t_2 \circ t_1$.

We have shown $t_1 \circ t_2 = t_2 \circ t_1 \ \forall t_1, t_2 \in T$.

Thus @ is commutative. We've shown $\Phi: \langle S, * \rangle \rightarrow \langle T, @ \rangle$ that commutativity of * implies commutativity of @. Therefore commutativity is a structural property.
A group is a set with a binary operation that is associative, has an identity element, and every element has an inverse.
A group is a set along with a binary operation that is associative, has an identity, and each element has an inverse.

A group is a set of numbers that have a relation between them and this relationship is associative, has an inverse, and has an identity. Each element has an inverse identity which is

A group is a set of elements that when a binary operation is applied to the elements, the binary operation is associative, two more

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A group is a set of elements that has an operation, which is associative, has an identity, and each element has an inverse.

A function with a binary operation that is associative and has an identity is called a group.

Each element in a group has an inverse.