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COG 356 tq- system

Abstract

These questions pertain to the tq- systems presented in GEB.

1. Write down the axiom schema and the three shortest axioms in the tq- system.

$xt-qx$, is an axiom, where x is a string of hyphens.

Axiom: $-t-q-$, $--t-q--$, $---t-q---$.

2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string:

$-----t-----q-----$.

Rule: Suppose that x , y , and z are all hyphen-strings. And suppose that $xtyqz$ is an old theorem.

Then, $xty-qzx$ is a new theorem.

$-----t-----q-----$

$5t6q10$

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.

$5*6 \neq 10$.

4. Working in M-mode, show that $-----t---q-----$ is a theorem in the tq- system.

$5*3 = 15$.

5. What are the two rules of the C- system?

RULE: Suppose x , y , and z are hyphen-strings. If $x-ty-qz$ is a theorem then Cz is a theorem.

RULE: Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

6. Working within the C- system, argue that $C-----$ is a theorem of the system.

$----t---q-----$ is a theorem in the tq- system, and can be represented as $xtyqz$, where z is a string of 8 hyphens, and so by the first rule in the C- system, $C-----$.

7. Does adding the following rule to the C- system constitute a Post production system for determining primes?

- Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

Please explain your response.

No, because the rule would require working outside the system.

8. First, please consider the following image of a quiche pan:

Second, recall that Hofstadter writes the following about positive space and negative space:

When a figure or “positive space” (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the “ground”, or “background”, or “negative space” - has also been drawn. According to this view, the quiche pan shown above, that I computationally rendered, would be considered negative space. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

The quiche pan generates the concept of negative space through the contrast with the border.

This is because the border gives off the effect of being the ground.

9. Consider the A- system as defined by the following axiom and rule:

- Axiom: A--
- Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax--.

Please answer the following questions with respect to the A-system:

(a) Show that A----- is a theorem of the A- system by working within the system.

Axiom: A- -

(1) A - - - - by rule 1

(2) A - - - - - by rule 1

(3) A - - - - - - by rule 1

(b) Specify a decision procedure for determining theorem-hood in the A- system.

If a string starts with an A, followed exclusively by an even number of hyphens, it is a theorem.

(c) Provide an I-mode argument that the string A----- is not a theorem of the A- system.

Since the string has an odd number of hyphens, it cannot be formed using valid rules, which only are able to add even numbers of hyphens.

(d) What subset of the natural numbers do you think it was my intent to capture with the A- system?

The even numbers.

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.

(a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B system.

For a theorem Ax in the A- system, where x is a string of hyphens, Ax- is a theorem in the B- system.

(b) Define a (valid) Post production system for the B- system in terms of one axiom and one rule.

Axiom: B-

Rule: Given the existing theorem Bx, where x is a string of hyphens, Bx- - is also a theorem.

(c) Derive B----- within the B- system.

Axiom: B-

(1) B- - - by rule 1

(2) B - - - - by rule 1

(3) B - - - - - by rule 1

(4) B - - - - - - by rule 1

(5) B - - - - - - - by rule 1

(d) What subset of the natural numbers does the B- system capture?

Odd numbers.

11. Under interpretation, what does the A- system theorem A----- say? Under interpretation, what does the B- system theorem B----- say?

It says “8,” and “11,” respectively.

12. According to Hofstadter, what does it mean for a set to be “recursively enumerable”? What does it mean for a set to be “recursive”?

To be recursively enumerable means that it can be generated according to typographical rules, and to be recursive means that a figure’s negative space can be treated as a figure itself.

13. Argue that the set of even numbers is recursively enumerable.

The set of all even numbers can be enumerated with the rule $f(n-1) + 2$.

14. Argue that the set of even numbers is recursive.

The “ground” of the set of all even numbers is the odd numbers, which is just as much of a figure as the even numbers.

15. Argue that the set of prime numbers is recursively enumerable.

The set of all prime numbers can be generated by considering all numbers whose only divisors are one and themselves.

16. Argue that the set of prime numbers is recursive.

The counterpart to the set of all prime numbers is the set of all composite numbers.

17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P----- within the P- system?

It seems like there isn't a nice rule for generating prime numbers.