GP - GEB Problem Set: The tq- System

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What's It All About?

This problem set is based on Chapter 3 of Hofstadter's GEB, in which he introduces the tq- system Just as the pq- system is isomorphic to both addition and subtraction, the tq- system is isomorphic to both multiplication and division. Like the problems posed for the pq- system, these problems focus on the basics of formal systems, such as axioms and theorems and rules of inference for producing theorems.

Task

Craft a nicely formatted document consisting of both the questions that you see below, and, immediately following each question, your answer to the question. Please format your work on this problem set in just the same way that you were asked to format the previous Post production system problem sets. And, as always, please save your document as a pdf file.

1. Write down the axiom schema and the three shortest axioms in the tq- system.

   Axiom schema: xt-qx is an axiom, whenever x is a hyphen-string.

   Three shortest axioms:

   - t - q -

   - - t - q - -

   - - - t - q - - -

2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string: -----t-----q-----

   Rule of inference: suppose that x, y, and z are all hyphen-strings. And suppose that xtyqz is an old theorem. Then, xty-qzx is a new theorem.

   - - - - - t - - - - - q - - - - - , x = - - - - - (5), y = - - - - - (5), z = - - - - - (5)

   - - - - - t - - - - - - q - - - - - - - - - - - , x = - - - - - (5), y = - - - - - - (6), zx = - - - - - - - - - - (5+5 = 10)

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.

   Since in order for the above to be in the tq- system, x has to start as 5 hyphens, and y has to start with 1 and this is a problem since when the rule is applied 5 times for y for to be 5, zx would be far past 10 hyphens, and it would never be - - - - - t - - - - - q - - - - -.

4. Working in M-mode, show that -----t---q--------------- is a theorem in the tq- system.
    1) - - - - - t - q - - - - - axiom
    2) - - - - - t - - q - - - - - - - - - - by rule of inference on (1)
    3) - - - - - t - - - q - - - - - - - - - - - - - - - by rule of inference on (2)
5. What are the two rules of the C- system?

    Rule: Suppose x,y, and z are hyphen-strings. If x-ty-qz is a theorem, then Cz is a theorem.

    Proposed Rule: Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

6. Working within the C- system, argue that C-------- is a theorem of the system.
    1) - - - - t - q - - - - axiom
    2) - - - - t - - q - - - - - - - - by rule of inference of tq- system on (1)
    3) C - - - - - - - - by rule of C- system on (2)

(I reread the question and realized it's not working within the C- system, it involved the tq- system?)

7. Does adding the following rule to the C- system constitute a Post production system for determining primes?
    • Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem. Please explain your response

    Adding the following rule to the C- system would not constitute a post production system because you can't manipulate the strings.

8. First, please consider the following image of a quiche pan:

    Second, recall that Hofstadter writes the following about positive space and negative space::

    When a figure or "positive space" (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the "ground", or "backgraound", or "negative space" - has also been drawn.

    According to this view, the quiche pan shown above, that I computationally rendered, would be considered negative space. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

    The quiche pan can be considered a negative space because the center is positive and the black area that forms the pan is in the background.

9. Consider the A- system as defined by the following axiom and rule:

    • Axiom: A--
    • Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax--.

Please answer the following questions with respect to the A-system:

(a) Show that A-------- is a theorem of the A- system by working within the system.

1) A - - axiom
2) A - - - - by rule on (1)
3) A - - - - - - by rule on (2)
4) A - - - - - - - - by rule on (3)

(b) Specify a decision procedure for determining theorem hood in the A- system.

Decision procedure for determining theorem hood in the A- system is by determining if there is an even number of hyphens.

(c) Provide an I-mode argument that the string A----------- is not a theorem of the A- system.

Since there is an odd number of hyphens, this can not be a theorem in the A- system since there is only one axiom, which includes two hyphens and the only rule is adding 2 hyphens, and even plus even can never be odd.

(d) What subset of the natural numbers do you think it was my intent to capture with the A- system?

Even numbers.

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.
    (a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B- system.
    Proposed Rule: Suppose x is a hyphen-string. If Bx is not a theorem then Cx is a theorem.
    (b) Define a (valid) Post production system for the B- system in terms of one axiom and one rule.
    Axiom: B-
    Rule Suppose x is a hyphen-string. If Bx is a theorem, then so is Bx- -.
    (c) Derive B----------- within the B- system.
    1) B - axiom
    2) B - - - by rule on (1)
    3) B - - - - - by rule on (2)
    4) B - - - - - - - by rule on (3)
    5) B - - - - - - - - - by rule on (4)
    6) B - - - - - - - - - - - by rule on (5)
    (d) What subset of the natural numbers does the B- system capture?
    Odd numbers.

11. Under interpretation, what does the A- system theorem A-------- say? Under interpretation, what does the B- system theorem B----------- say?

A - - - - - - - - exist in the A- system, which means 8(the number of hyphens) is an even number. While B- - - - - - - - - - - exists in the B- system, which means 11(the number of hyphens) is an odd number.

12. According to Hofstadter, what does it mean for a set to be “recursively enumerable”? What does it mean for a set to be “recursive”?

    Hofstadter says for a set to be recursively enumerable, it can be generated according to typographical rules. A set is “recursive” if it is recursively enumerable and it’s complement is also recursively enumerable.

13. Argue that the set of even numbers is recursively enumerable.

    The set of even numbers is recursively enumerable because it can be generated from typographical rules (A- system).

14. Argue that the set of even numbers is recursive.

    The set of even numbers is recursive because it is recursively enumerable and it’s complement (odd numbers) is also recursively enumerable (B- System).

15. Argue that the set of prime numbers is recursively enumerable.

    The set of prime numbers is recursively enumerable because it can be generated from typographical rules.

16. Argue that the set of prime numbers is recursive.

    The set of prime numbers is recursive because it is recursively enumerable and it’s complement (composite numbers) is also recursively enumerable.

17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P----- within the P- system?

    Because the P- system does not have any typographical rules, instead it is reliant on the C- system, and Cx to be non-theorem.