GP - GEB Problem Set: The tq- System

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Abstract: This problem set is based on Chapter 3 of Hofstadter's GEB, which he introduces the tq- system. Just as the pq- system is isomorphic to both addition and subtraction, the tq- system is isomorphic to both multiplication and division. Like the problems posed for the pq- system, these problems focus on the basics of formal systems, such as axioms and theorems and rules of inference for producing theorems.

Tasks

1. Write down the axiom schema and the three shortest axioms in the tq- system.

   Axiom Schema: *x*t-q*x* is an axiom, whenever *x* is a hyphen string.

   Three Shortest Axioms:

   (1) - t - q -

   (2) - - t - q - -

   (3) - - - t - q - - -

2. Write down the sole rule of inference for the tq-system and apply it to the well-formed string: -----t-----q-----.

   Rule of Inference: Suppose that *x, y,* and *z* are all hyphen-strings. And suppose that *x*t*y*q*z* is an old theorem. Then, *x*t*y*-q*zx* is a new theorem.

   (1). - - - - - t - - - - - q - - - - -

   (2). - - - - - t - - - - - - q - - - - - - - - - -

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.

I noticed that the length of the hyphens were the same in (1), meaning the first two sets of strings couldn't possibly equal the third through multiplication. The third string would need to be significantly longer for both (1) and (2) if they were working theorems in the tq- system.

4. Working in M-mode, show that - - - - - t - - - q - - - - - - - - - - - - - - - is a theorem in the tq- system.

Top-Down procedure

(1) - - - - - t - - - q - - - - - - - - - - - - - - -

(2) - - - - - t - - q - - - - - - - - - -

(3) Axiom: - - - - - t - q - - - - -

5. What are the two rules of the C- system?

<u>tq- System Rule of Inference:</u> Suppose that *x, y,* and *z* are all hyphen-strings. And suppose that '*x*t*y*q*z*' is an old theorem. Then, '*x*t*y* - q*zx*' is a new theorem.

<u>C-type Theorem Rule:</u> Suppose *x, y,* and *z* are hyphen-strings. If '*x* - t*y* - q*z*' is a theorem, then C*z* is a theorem.

6. Working within the C - system, argue that C - - - - - - - - is a theorem of the system.

(1) Axiom: - - t - q - -

(2) - - t - - q - - - - from (1) by the tq- system rule of inference

(3) - - t - - - q - - - - - - from (2) by the tq- system rule of inference

(4) - - t - - - - q - - - - - - - - from (3) by the tq- system rule of inference

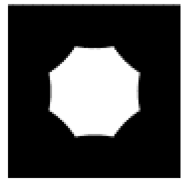
(5) C - - - - - - - - from (4) by the C-type theorem rule

7. Does adding the following rule to the C- system constitute a post-production system for determining primes?

- Suppose *x* is a hyphen-string. If C*x* is **not** a theorem, then P*x* **is** a theorem.

No. This rule is proposing that the operations go outside of the system to check for a hypothetical "Table of Nontheorems". The rule is non-typographical and needs to be dropped.

8. First, please consider the following image of a quiche pan:



Second, recall that Hofstadter writes the following about positive space and negative space:

When a figure or "positive space" (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape – also called the "ground", or "background", or "negative space" – has also been drawn. According to this view, the quiche pan shown above, that I computationally rendered, would be considered **negative space**. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

Because the frame and background are white, the "positive space" being drawn in the image is actually outlining the quiche pan. Because the positive space has been drawn, the negative space has also been drawn, creating the pan out of the background.

9. Consider the A- system as defined by the following axiom and rule:

- Axiom: A- -
- Rule: Suppose that *x* is a hyphen-string. If A*x* is a theorem, so is A*x* - - .

Please answer the following questions with respect to the A- system:

(a) Show that A - - - - - - - - (8) is a theorem of the A- system by working within the system.

(1) Axiom: A - -
(2) A - - - - from (1) by the only rule
(3) A - - - - - - from (2) by the only rule
(4) A - - - - - - - - from (3) by the only rule

(b) Specify a **decision procedure** for determining theoremhood in the A- system

If the string starts with 'A' followed by two hyphens, or a number of hyphens divisible by two.

(c) Provide an I-mode argument that the string A - - - - - - - - - - - (11) is not a theorem of the A- system.

Because there are an odd number of dashes, it isn't possible for this to be a theorem in the A- system.

(d) What subset of the natural numbers do you think it was my intent to capture with the A- system?
Even numbers

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- system does not capture.

(a) Propose, by analogy with the rule of page 66 of GEB, an invalid rule for producing theorems in the B- system.

Suppose *x* is a hyphen-string. If A*x* is *not* a theorem, then B*x* *is* a theorem.

(b) Define a (valid) post-production system for the B- system in terms of one axiom and one rule.

- **Axiom: B –**
- **Rule: Suppose that *x* is a hyphen-string. If B*x* is a theorem, so is B*x* - -.**

(c) Derive B - - - - - - - - - - - (11) within the B- system.

(1) Axiom: B -
(2) B - - - from (1) by the only rule
(3) B - - - - - from (2) by the only rule
(4) B - - - - - - - from (3) by the only rule
(5) B - - - - - - - - - from (4) by the only rule
(6) B - - - - - - - - - - - from (5) by the only rule

(d) What subset of the natural numbers does the B- system capture?

Odd numbers

11. Under **interpretation**, what does the A- system theorem A - - - - - - - - (8) say? Under **interpretation,** what does the B- system theorem B - - - - - - - - - - - (11) say?

A - - - - - - - - represents the number 8, while B - - - - - - - - - - - represents the number 11

12. According to Hofstadter, what does it mean for a set to be "recursively enumerable"? What does it mean for a set to be "recursive"?

For a set of strings to be "recursively enumerable", it means that it can be generated according to typographical rules (the set of theorems of a formal system).

For a set to be "recursive", the set itself needs to be recursively enumerable, but its complement is also recursively enumerable.

13. Argue that the set of even numbers is recursively enumerable.

The set of even numbers is recursively enumerable because they are generated typographically within the rules of the A- system.

14. Argue that the set of even numbers is recursive.

The set of even numbers is recursive because the set itself is recursively enumerable, and its counterpart (odd number set), is recursively enumerable within the B- system.

15. Argue that the set of prime numbers is recursively enumerable.

The set of prime numbers is recursively enumerable because they can be generated typographically by using the axiom schema and rules within the *does not divide* procedure, and the rules within the *divisor-freeness* procedure.

<u>Axiom Schema</u>: '*xy*DND*x*' where *x* and *y* are hyphen-strings.

<u>Rule</u>: If '- - DND*z*' is a theorem, so is '*x*DND*xy*'.

<u>Rule</u>: If '- - DND*z*' is a theorem, so is '*z*DF - -'.

<u>Rule</u>: If '*z*DF*x*' is a theorem and also '*x* – *DND*z' is a theorem, then '*z*DF*x* -' is a theorem.

<u>Rule</u>: If '*z* – DF*z*' is a theorem, then P*z*- is a theorem.

16. Argue that the set of prime numbers is recursive

The set of prime numbers themselves were proven to be recursively enumerable within the previous question, and we know that its counterpart (composite set) can be typographically generated through the rules of the tq- system and the rule defining C-type theorems. Since both counterparts are recursively enumerable, that makes the set of prime numbers recursive.

17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P - - - - - within the P- system?