GP - GEB Problem Set: The tq- System

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Abstract: This problem set is based on Chapter 3 of Hofstadter's GEB, in which he introduces the tq- system Just as the pq- system is isomorphic to both addition and subtraction, the tq- system is isomorphic to both multiplication and division. Like the problems posed for the pq- system, these problems focus on the basics of formal systems, such as axioms and theorems and rules of inference for producing theorems.

Task

1. Write down the axiom schema and the three shortest axioms in the tq- system.

Schema: *x* t – q *x* is an axiom, whenever x is a hyphen-string.

1) - - t – q – -
2) – - - t - q - - -
3) – - - - t - q - - - -

2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string:

-----t-----q-----.

Rule of inference: Suppose that x, y, and z are all hyphen-strings. And suppose that xtyqz is an old theorem. Then, xty-qzx is a new theorem.

- - - - - t - - - - - - q ------------------------------

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.

If the answer above was right, then I would take a guess and say that it's not a theorem because Cz would be a prime number instead of composite which is what I had.

4. Working in M-mode, show that -----t---q--------------- is a theorem in the tq- system.

1) - - t – q - - 3) - - - - t - q - - - - 5) - - - - - t - - q - - - - - - - - - -

2) - - - t - q - - - 4) - - - - - t – q - - - - - 6) - - - - - t - - - q - - - - - - - - - - - - - - -

Or

It's a theorem because X which is five times Y with the value of three gives you fifteen, equal to the number of hyphens at Z.

5. What are the two rules of the C- system?

Rule: Suppose x, y, and z are hyphen-strings. If x – ty - qz is a theorem, then Cz is a theorem.

Rule: Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

6. Working within the C- system, argue that C-------- is a theorem of the system.

C - - - - (4)

C - - - - - - (6) Or It's a theorem because there's eight hyphens (composite number).

C - - - - - - - - (8)

7. Does adding the following rule to the C- system constitute a Postproduction system for determining primes? • Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem. Please explain your response:

I think that it may constitute a post-production system because this rule sets up clarity and somewhat of a formal structure as to what is an what is not a theorem.

8. First, please consider the following image of a quiche pan: Second, recall that Hofstadter writes the following about positive space and negative space: When a figure or "positive space" (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the "ground", or "background", or "negative space" - has also been drawn. According to this view, the quiche pan shown above, that I computationally rendered, would be considered negative space. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

The Quiche pan in the image represents a negative space because it is not a distinguishable form and serves more like a background to the positive space which does have a shape or form.

9. Consider the A- system as defined by the following axiom and rule:

• Axiom: A--

• Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax--. Please answer the following questions with respect to the A-system:

(a) Show that A-------- is a theorem of the A- system by working within the system.

(1) A - -

(2) A - - - -

(3) A - - - - - -

(4) A - - - - - - - -

(b) Specify a decision procedure for determining theorem hood in the A- system.

You can check for theorem hood by checking the total number of hyphens divisibility.

(c) Provide an I-mode argument that the string A----------- is not a theorem of the A- system.

It's not a theorem because it's not divisible by two.

(d) What subset of the natural numbers do you think it was my intent to capture with the A-system?

Numbers divisible by two.

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.

(a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the Bsystem.

Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

(b) Define a (valid) Post produduction system for the B- system in terms of one axiom and one rule.

Axiom: B -

Rule: Suppose x is a string of hyphens, if Bx is a theorem so Bx - - is a theorem.

(c) Derive B----------- within the B- system.

(1) B -

(2) B - - -

(3) B - - - - -

(4) B - - - - - - -

(5) B - - - - - - - - -

(6) B - - - - - - - - - - -

(d) What subset of the natural numbers does the B- system capture?

It would be prime numbers so 2k+ 1 where k is any integer.

11. Under interpretation, what does the A- system theorem A-------- say? Under interpretation, what does the B- system theorem B----------- say?

It says that A - - - - - - - - is a theorem of the A- system because eight, the number of hyphens is divisible by 2. Its complement B - - - - - - - - - - - - says that it is a theorem of the B- system because eleven the number of hyphens is not divisible by two.

12. According to Hofstadter, what does it mean for a set to be "recursively enumerable"? What does it mean for a set to be "recursive"?

Recursively enumerable represents a set produced according to typographical rule while Recursive refers to a figure whose ground represents another figure. Meaning its complement set is also recursively enumerable

13. Argue that the set of even numbers is recursively enumerable.

A set of even numbers is recursively enumerable because you can prove this being testing their divisibility. Therefore, a typographical decision procedure set up as such.

14. Argue that the set of even numbers is recursive.

They are recursive because both even and odd numbers are recursively enumerable

15. Argue that the set of prime numbers is recursively enumerable.

The set of prime numbers is recursively enumerable because it is possible to have typographical rules that say the set consist of numbers not divisible by two and greater than one.

16. Argue that the set of prime numbers is recursive.

Prime numbers are recursively enumerable, and its complement composite numbers are also recursively enumerable.

17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P----- within the P- system?

It may have to do with the fact that since P – consist of checking the divisibility for prime numbers.