1. Write down the axiom schema and the three shortest axioms in the tq- system.
xt-qx is an axiom, whenever x is a hyphen string
-t-q-
- t - - q -
- - t - q -
2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string: -----t-----q-----.

Suppose that x, y, and z are all hyphen-strings. And suppose that
x ty qz is an old theorem. Then, xty-qzx is a new theorem.

-----t-----q-----
-----t---- - -q-----

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.
The tq system is supposed to represent multiplication, but the above string does not do so, so it is not a theorem.
4. Working in M-mode, show that -----t---q---------- is a therem in the tq- system.
- - - - - t - q - - - - -
- - - - - t - - q - - - - - - - - - -
- - - - - t - - - q - - - - - - - - - - - - - - -
5. What are the two rules of the C- system?
Suppose that x, y, and z are all hyphen-strings. And suppose that
x ty qz is an old theorem. Then, xty-qzx is a new theorem.
Suppose x, y, and z are hyphen-strings. If x-ty-qz is a theorem then C z is a theorem.

6. Working within the C- system, argue that C-------- is a theorem of the system.
- - - - t - q - - - -
- - - - t - - q - - - - - - - -
C- - - - - - -
7. Does adding the following rule to the C- system constitute a Post production system for determining primes?
• Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.
Please explain your response
It would not, because a formal typographical system does not contain the ability to determine if a string is a theorem.
8. First, please consider the following image of a quiche pan:
Second, recall that Hofstadter writes the following about positive space and negative space::
When a figure or “positive space” (e.g., a human form, or a letter, or a still life) is drawn inside a

frame, an unavoidable consequence is that its complementary shape - also called the “ground”, or “background”, or “negative space” - has also been drawn. According to this view, the quiche pan shown above, that I computationally rendered, would be considered negative space. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

This would be considered negative space, because the figure that is drawn is the outside borders of the quiche pan, rather than the pan itself.

9. Consider the A- system as defined by the following axiom and rule:
• Axiom: A--
• Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax--.
Please answer the following questions with respect to the A-system:
(a) Show that A-------- is a theorem of the A- system by working within the system.
A - -
A - - - -
A - - - - - -
A - - - - - - - -
(b) Specify a decision procedure for determining theoremhood in the A- system.
Is X, the number of hyphens in x, divisible by 2? If so it is a theorem
(c) Provide an I-mode argument that the string A---------- is not a theorem of the A- system.
It is not even, therefore it is not divisible by 2, therefore it is not a theorem.
(d) What subset of the natural numbers do you think it was my intent to capture with the A-system?
Even numbers.

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.
(a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B- system.
If x is a hyphen string from Ax, then Bx is a theorem.
(b) Define a (valid) Post produduction system for the B- system in terms of one axiom and one rule.
Axiom: B-
Rule: Suppose x is a hyphen-string. If Bx is a theorem, so is Bx- -
(c) Derive B---------- within the B- system.
B -
B - - -
B - - - - -

B - - - - - - -
B - - - - - - - - -
B- - - - - - - - -
(d) What subset of the natural numbers does the B- system capture?
Odd numbers.

11. Under interpretation, what does the A- system theorem A-------- say? Under interpretation, what does the B- system theorem B---------- say?
X, the number of hyphens in string x of Ax, is even.
X, the number of hyphens in string x of Bx, is odd.
12. According to Hofstadter, what does it mean for a set to be "recursively enumerable"? What does it mean for a set to be "recursive"?
A set is rescursively enumerable when its absent numbers are an accidental byproduct of the inclusion of that sets numbers.
A set is recursive when its absent numbers form a set.
13. Argue that the set of even numbers is recursively enumerable.
The set of even numbers is recursively enumerable because its absent numbers form the set of odd numbers only by merit of the fact that if a number is not even, it is odd.
14. Argue that the set of even numbers is recursive.
The set of even numbers is recursively enumerable because its absent numbers form the set of odd numbers.
15. Argue that the set of prime numbers is recursively enumerable.
The set of prime numbers is recursively enumerable because its absent numbers form the set of composite numbers only by merit of the fact that if a number is not composite, it is prime.
16. Argue that the set of prime numbers is recursive.
The set of prime numbers is recursively enumerable because its absent numbers form the set of composite numbers.
17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like P----- within the P- system?
Because the p- system does not have a decision procedure.