Name: Amanda Pirie

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Problem Set: The tq-System

Abstract: This problem set is based on Chapter 3 of Hofstadter's GEB, in which he introduces the tq- system Just as the pq- system is isomorphic to both addition and subtraction, the tq-system is isomorphic to both multiplication and division. Like the problems posed for the pq-system, these problems focus on the basics of formal systems, such as axioms and theorems and rules of inference for producing theorems

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The 17 Questions and Answers

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1. Write down the axiom schema and the three shortest axioms in the tq- system.

Axiom schema: xt-qx is an axiom, whenever x is a hyphen string.

- t - q - , - - t - q - - , - - - t - q - - -

2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string: - - - - - t - - - - - q - - - - -.

Rule of inference: Suppose that x, y,and z are all hyphen-strings. And suppose that xtyqz is an old theorem. Then, xty-qzx is a new theorem.

Before rule of inference: - - - - - t - - - - - q - - - - -
After rule of inference: - - - - - t - - - - - - q- - - - - - - - - -

3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.

5 hyphens * 5 hyphens is not 10 hyphens. If it were to be a theorem in the tq- system there would be 25 hyphens on the end

(beginning # of hyphens * middle # of hyphens = end # of hyphens)

4. Working in M-mode, show that - - - - - t - - - q - - - - - - - - - - - - - - - is a theorem in the tq- system.

5 hyphens * 3 hyphens = 15 hyphens

(beginning # of hyphens * middle # of hyphens = end # of hyphens)

5. What are the two rules of the C- system?

RULE: Suppose x, y, and z are hyphen-strings. If x-ty-qz is a theorem then C z is a theorem.

PROPOSED RULE: Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem.

6. Working within the C- system, argue that C-------- is a theorem of the system.

Suppose we have theorem - - t - - - - q - - - - - - - - then C - - - - - - - - is a theorem.

7. Does adding the following rule to the C- system constitute a Post production system for determining primes?

Suppose x is a hyphen-string. If Cx is not a theorem, then Px is a theorem. Please explain your response

It does not constitute a post production system because checking if Cx is not a theorem is not explicitly a typographical operation.

“In fact, our aim is to find just such a system. But the Proposed Rule is not a typographical rule, and must be dropped.”

8. First, please consider the following image of a quiche pan:

Second, recall that Hofstadter writes the following about positive space and negative space:

When a figure or “positive space” (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the “ground”, or “background”, or “negative space” - has also been drawn.

According to this view, the quiche pan shown above, that I computationally rendered, would be considered negative space. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

The quiche pan was drawn in white on a black canvas, where the white color was viewed as a mark on the canvas itself; just as if the colors on the canvas were reversed. If the canvas were white and the quiche pan was black, it may be viewed as a mark.

The image was rendered so that the quiche pan was considered negative space because the quiche pan would be considered positive while the black part would be considered the “background”

9. Consider the A- system as defined by the following axiom and rule:

- Axiom: A--
- Rule: Suppose that x is a hyphen-string. If Ax is a theorem, so is Ax–

Please answer the following questions with respect to the A-system:
(a) Show that A-------- is a theorem of the A- system by working within the system.

A - -	Axiom
A - - - -	Rule (1)
A - - - - - -	Rule (1)
A - - - - - - - -	Rule (1)

(b) Specify a decision procedure for determining theorem hood in the A- system.

If the number of hyphens after A is divisible by 2, then the string in question is a theorem

(c) Provide an I-mode argument that the string A - - - - - - - - - - - is not a theorem of the A-system.

The amount of hyphens after A is odd, therefore the string is not a theorem.

(d) What subset of the natural numbers do you think it was my intent to capture with the A-system?

Even natural numbers

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture

(a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B- system.

Suppose x is a hyphen-string. If Ax is not a theorem, then Bx is a theorem.

(b) Define a (valid) Post production system for the B- system in terms of one axiom and one rule.

Axiom: B -
Rule: Suppose that x is a hyphen-string. If Bx is a theorem, so is Bx - -

(c) Derive B---------- within the B- system.

B -	Axiom
B - - -	Rule (1)
B - - - - -	Rule (1)
B - - - - - - -	Rule (1)
B - - - - - - - - -	Rule (1)
B - - - - - - - - - -	Rule (1)

(d) What subset of the natural numbers does the B- system capture?

The B- system captures the odd natural numbers

11. Under interpretation, what does the A- system theorem A-------- say? Under interpretation, what does the B- system theorem B---------- say?

Under interpretation, A - - - - - - - - demonstrates that “8 is even”
Under interpretation, B - - - - - - - - - - demonstrates that “11 is odd”

12. According to Hofstadter, what does it mean for a set to be “recursively enumerable”? What does it mean for a set to be “recursive”?

If a set is recursively enumerable, the set can be generated. If a set is recursive, the set can be generated according to its negation or non-negation.

13. Argue that the set of even numbers is recursively enumerable.

The addition of two to a value of zero would generate an even.

14. Argue that the set of even numbers is recursive.

The addition of two to a value of one would generate an odd. What's not generated by that rule is even.

15. Argue that the set of prime numbers is recursively enumerable.

The set of prime numbers is proven recursively enumerable through Hofstadter's C- system.

16. Argue that the set of prime numbers is recursive.

Since the composite numbers can be generated, the set of prime numbers is recursive. Everything that isn't in the composite numbers is prime (hence the negation of the composite).

17. In a sentence or two, explain why you think that I am not asking you in this problem to derive something like P----- within the P- system?

Deriving P - - - - - would be too time consuming, therefore you would not ask us to do so.