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PQ- system

Abstract

This paper contains 21 questions regarding the second chapter of Goedel, Escher, Bach.

1. What is the formal system of Chapter 2 called?

The pq- system.

2. What are the distinct symbols of this formal system?

P, Q, and -.

3. How many axioms in the pq- system?

Infinite.

4. Write down the axiom schema for the pq- system.

xp-q-x is an axiom, where x is composed only of hyphens.

5. Write down the three shortest axioms in the pq- system.

 -p-q- -, - -p-q- - -, and - - -p - q- - - -.

6. Write down the sole rule of production for the pq- system.

RULE 1: Suppose x, y, and z are strings containing only hyphens, and xpyqz is known to be a theorem, then xpy-qz- is a theorem.

7. Show that --p---q----- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

(1) --p-q--- axiom

(2) --p--q---- by rule 1

(3) --p---q----- by rule 1

8. Show that -----p----q--------- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

(1) -----p-q----- axiom

(2) -----p--q------- by rule 1

(3) -----p---q-------- by rule 1

(4) -----p----q--------- by rule 1

9. Write down a string of symbols in the pq- system which is not well formed.

qp

10. State a decision procedure for the pq- system.

The first two hyphen groups should add up to the third group of hyphens.

11. In the longest paragraph on page 48, Hofstadter engages in some "top-down" reasoning. In one sentence, articulate exactly what it is that he demonstrates with his top-down reasoning in this paragraph?

Hofstadter explains that formal systems are more interesting when they have both lengthening and shortening rules, since it leads to more indeterminism.

12. In one sentence, characterize "top-down" reasoning.

Top-down reasoning is working from a specific case to get down to an axiom.

13. In one sentence, characterize "bottom-up" reasoning.

Bottom-up reasoning is working from an axiom up to a specific case.

14. Consider the procedure for generating theorems of the pq- system given at the top of page 49. What will be in the bucket after executing statement (1a)? After (1b)? After (2a)? After (2b)? After (3a)? After (3b)?

-p-q- -, - -p-q- - -, - - -p-q- - - -, -p- -q - - -, - -p - -q - - - -, and - - -p - -q - - - - -.

15. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pq system and related matters? Answer in just one sentence!

It shows that every theoremhood of any string of length n can be proven in fewer than n steps.

16. What is an isomorphism?

An isomorphism is a mapping between two complex structures, such that each part of each structure corresponds to a specific part in the other.

17. What is an interpretation in the context of a formal system?

An interpretation is a mapping from a symbol to a word.

18. When was Linear B deciphered?

Linear B was deciphered some time before *The Decipherment of Linear B* was published in 1958.

19. How many meaningful interpretations of the pq- system did Hofstadter present in this chapter?

Two.

20. How many meaningless presentations of the pq- system are there?

One.

21. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled "Formal Systems and Reality".

Hofstadter posits that reality might be able to be modeled as a formal system, with the arrangement of particles at the beginning of time representing an axiom, and each successive state in time being a theorem.