

GP - GEB Problem Set: The pq- System

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What's It All About?

This problem set is based on Chapter 2 of Hofstadter's GEB. In it he presents the first of his Post productions systems that have a numeric flavor. The sequence of production systems that Hofstadter presents, particularly the numeric flavored systems, pave the way for his discussion of Godel's theorem.

Task

Craft a nicely formatted document consisting of both the questions that you see below, and, immediately follow- ing each question, your answer to the question. Please format your work on this problem set in just the same way that you were asked to format the MIU System problem set. And, as always, please save your document as a pdf file.

1. What is the formal system of Chapter 2 called?

The pq- System.

2. What are the distinct symbols of this formal system?

p q -

3. How many axioms in the pq- system?

The pq- system has an infinite number of axioms.

4. Write down the axiom schema for the pq- system.

xp-qx- is an axiom, whenever x is composed of hyphens only.

5. Write down the three shortest axioms in the pq- system.

- p - q - -

-- p - q - - -

--- p - q - - - -

6. Write down the sole rule of production for the pq- system.

Rule: Suppose x, y, and z all stand for particular strings containing only hyphens. And suppose that xpyqz is known to be a theorem. Then xpy-qz- is a theorem.

7. Show that $--p--q----$ is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

1) $--p-q---$ axiom

2) $--p--q-----$ rule applied on (1)

3) $--p----q-----$ rule applied on (2)

8. Show that $----p----q-----$ is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

1) $-----p-q-----$ axiom

2) $-----p--q-----$ rule applied on (1)

3) $-----p---q-----$ rule applied on (2)

4) $-----p----q-----$ rule applied on (3)

9. Write down a string of symbols in the pq- system which is not well formed.

$--p--p--p--q-----$

10. State a decision procedure for the pq- system.

The first two hyphen-groups should add up, in length, to the third hyphen-group.

11. In the longest paragraph on page 48, Hofstadter engages in some “top-down” reasoning. In one sentence, articulate exactly what it is that he demonstrates with his top-down reasoning in this paragraph?

He demonstrates with his top down reasoning that you can work backwards to show if there exists an axiom that is a part of the pq- system.

12. In one sentence, characterize “top-down” reasoning.

Top down reasoning is when you start with the theorems to find the axiom.

13. In one sentence, characterize “bottom-up” reasoning.

Bottomup reasoning is when you start with the axiom, and produce the theorems.

14. Consider the procedure for generating theorems of the pq- system given at the top of page 49. What will be in the bucket after executing statement (1a)? After (1b)? After (2a)? After (2b)? After (3a)? After (3b)?

1a) - p - q - -

1b) - p - q - - and - p - - q - - -

2a) - p - q - - and - p - - q - - - and - - p - q - - -

2b) - p - q - - and - p - - q - - - and - - p - q - - - and - p - - q - - - and - p - - - q - - - - and - - p - - q - - - -

3a) - p - q - - and - p - - q - - - and - - p - q - - - and - p - - q - - - and - p - - - q - - - - and - - p - - q - - - - and - - - p - q - - - -

3b) - p - - q - - - and - p - - - q - - - - and - - p - - q - - - - and - p - - - q - - - - and - p - - - - q - - - - - and - - p - - - - q - - - - - and - - - p - - q - - - - -

15. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pq- system and related matters? Answer in just one sentence!

You can test whether something is a part of the pq- system by using the decision procedure of either top down or bottom up.

16. What is an isomorphism?

Isomorphism was defined as an information preserving transformation. Hofstadter adds on and said that isomorphism applies when two complex structures can be mapped onto each other, in such a way that to each part of one structure there is a corresponding part in the other structure, where "corresponding" means that the two parts play similar roles in their respective structures.

17. What is an interpretation in the context of a formal system?

An interpretation is a symbol-word correspondence.

18. When was Linear B deciphered?

Linear B was deciphered in *The Decipherment of Linear B* by John Chadwick.

19. How many meaningful interpretations of the pq- system did Hofstadter present in this chapter.

2, one plus two equals three (page 50) and two equals three taken from 5 (page 52).

20. How many meaningless presentations of the pq- system are there?

Infinite amount.

21. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled "Formal Systems and Reality".

Reality itself can possibly be a very complicated formal system, whose symbols are in a three dimensional vacuum and of which everything is composed of. If this is true then the axiom would be the "beginning of time". The rules in this formal system would be the laws of physics allowing us to produce the "next" instant.