GP - GEB Problem Set: The pq- System

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Abstract: This problem set is based on Chapter 2 of Hofstadter's GEB. In this chapter, Hofstadter describes a post-production system called the pq- system. Axiom schemas and decision procedures are introduced, and examples are given through the use of the pq- system. Top-down and Bottom-up reasoning are described, and examples of each procedure are given within the constraints of the pq-system. Isomorphism is both defined and explained in regard to formal systems, and both *meaningful* and *meaningless* interpretations of the pq- system are listed. Hofstadter sets the stage for more complex formal systems in chapters to come.

Ten Salient Ideas from "Introduction: A Musico-Logical Offering"

1. What is the formal system of Chapter 2 called?
   The formal system for chapter 2 is called the pq- system.
2. What are the distinct symbols of this formal system?
   p, q, and the hyphen.
3. How many axioms in the pq- system?
   There are an infinite number of axioms in the pq- system.
4. Write down the *axiom schema* for the pq-system.
   xp-qx-
5. Write down the three shortest axioms in the pq- system.
   a. **-p-q--**
   b. **-p--q---**
   c. **-p---q----**
6. Write down the sole production of the pq system.
   Suppose x, y, and z all stand for particular strings containing only hyphens. And suppose that xpyqz is known to be a theorem. Then xpy-qz is a theorem.
7. Show that --p---q----- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.
   (1) --p-q--- axiom
   (2) --p--q---- from (1) by the only rule
   (3) --p---q----- from (2) by the only rule

8. Show that -----p----q--------- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.
   (1) -----p-q------ axiom
   (2) -----p--q------- from (1) by the only rule
   (3) -----p---q-------- from (2) by the only rule
   (4) -----p----q--------- from (3) by the only rule
9. Write down a string of symbols in the pq- system which is *not* well formed.

-p-p-q

10. State a *decision procedure* for the pq- system
If the string starts with a group of hyphens and then a 'p', followed by a group of hyphens and then a q, then followed by another group of hyphens, and the length of the last group of hyphens equals the sum of the lengths of the first two hyphen groups, then it is a well-formed string and is a theorem.
11. In the longest paragraph on page 48, Hofstadter engages in some "top-down" reasoning. In one sentence articulate exactly what it is that he demonstrates with his top-down reasoning in this paragraph?
He demonstrates that if a formal system has only lengthening rules, then it needs to have a decision procedure.
12. In one sentence, characterize "top-down" reasoning.
Top-down reasoning starts by checking the given string to see if it is an axiom, and if not, go over the rules one by one and reduce the problem down by checking for shorter possible strings/theorems, and repeating this process until the axiom is found.
13. In one sentence, characterize "bottom-up" reasoning.
Bottom-up reasoning involves starting with the simplest possible axioms and tests those axioms by throwing it into a "bucket" and applying the rule of inference, putting that result back into the bucket, and if the string you are testing for shows up in the process, it's a theorem.
14. Consider the procedure for generating theorems of the pq-system given at the top of page 49. What will be in the bucket after executing statements (1a) and (1b) and (2b) and (3a) and (3b)
(1a) -p-q--
(2b) -p--q---
(2a) -p--q---
(2b)-p---q----
(3a)-p---q----
(3b)-p----q-----
15. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pq-system and related matters?
The decision procedure given on the top of page 49 is an example of a bottom-up procedure.
16. What is an *isomorphism*?
When two complex structures can be mapped onto each other, and to each part of one structure, there is a corresponding part in the other structure.
17. What is an *interpretation* in the context of a formal system?
Interpretation is the correspondence between true statements and interpreted theorems.
18. When was Linear B deciphered?
1952
19. How many *meaningful* interpretations of the pq- system did Hofstadter present in this chapter?
2
20. How many *meaningless* presentations of the pq- system are there?
1
21. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled, "Formal Reality".
Hofstadter talks about how a formal system and the part of reality it is mimicking are independent from each other. He mentions that with these formal systems, we learn something about the nature of that process and how it is mimicked by symbols. He then dives into whether all of reality could be a formal system or not, and whether or not the universe operates deterministically.