GP - GEB Problem Set: The pq- System

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Abstract: This problem set is based on Chapter 2 of Hofstadter's GEB. In it he presents the first of his Post productions systems that have a numeric flavor. The sequence of production systems that Hofstadter presents, particularly the numeric flavored systems, pave the way for his discussion of Godel's theorem.

Task

1. What is the formal system of Chapter 2 called?

Pq-system.

2. What are the distinct symbols of this formal system?

P → Q → - (hyphen)

3. How many axioms in the pq- system?

Infinite number of axioms.

4. Write down the axiom schema for the pq- system.

Schema: x P – Q x – is an axiom, whenever x is composed of hyphens only.

5. Write down the three shortest axioms in the pq- system.

- p – q –

- - p – q - - -

- - - p – q - - - -

6. Write down the sole rule of production for the pq- system.

Rule: Suppose x, y, and z all stand for strings containing only hyphens. And suppose that xPyQz is known to be a theorem. Then x P y – Q z – is a theorem.

7. Show that --p---q----- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

This is a theorem because the two hyphens behind the p and three Infront is equivalent to the five hyphens in front of q.

8. Show that -----p----q--------- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

This is a theorem because the hyphens before q are equal to nine just as the ones after it are.

9. Write down a string of symbols in the pq- system which is not well formed.

- p – p – q - - -

10. State a decision procedure for the pq- system.

Hofstadter's mentions that the pq- system only has lengthening rules as opposed to ones that shorten the string. So, a decision procedure that is mentioned is when something is not an axiom. There aren't shortening rules, but you can shorten the string intentionally to figure out where it broke an axiom.

11. In the longest paragraph on page 48, Hofstadter engages in some "top-down" reasoning. In one sentence, articulate exactly what it is is that he demonstrates with his top-down reasoning in this paragraph?

Hofstadter's demonstrating that checking a theorem can be done working backwards from an already lengthened string.

12. In one sentence, characterize "top-down" reasoning.

Top-down reasoning depicts a decision procedure in which you uncover the creation of the string by working on it backwards.

13. In one sentence, characterize "bottom-up" reasoning.

Bottom-up reasoning is a representation of decision procedures starting with the axiom and working its way up.

14. Consider the procedure for generating theorems of the pq- system given at the top of page 49. What will be in the bucket after executing statement (1a)? After (1b)? After (2a)? After (2b)? After (3a)? After (3b)?

15. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pqsystem and related matters? Answer in just one sentence!

I think that Hofstadter's intention may be to show the reader that the pq- system is not solely about its lengthening rules although that is all that is allowed. Working top to down provides answers as well.

16. What is an isomorphism?

Isomorphism is when two complex forms have relations to each other and perform similar actions.

17. What is an interpretation in the context of a formal system?

Interpreting something involves learning about it. In the context of a formal system, interpretation can mean knowing the axioms, what the system can produce, decision procedures and even the slightest bit of isomorphism.

18. When was Linear B deciphered?

1952

19. How many meaningful interpretations of the pq- system did Hofstadter present in this chapter.

Five.

20. How many meaningless presentations of the pq- system are there?

One.

21. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled "Formal Systems and Reality".

In this section of the textbook Hofstadter's starts to mention what the purpose of the pq- system is. He states that through the pq- system we haven't learned anything new concerning addition properties. But Something that can be taken away from this is the isomorphism between formal systems and reality.