Daniel Petti

1. What is the formal system of Chapter 2 called?

The p-q system.

2. What are the distinct symbols of this formal system?

p, q, -

3. How many axioms in the pq- system?

There are an infinite number of axioms.

4. Write down the axiom schema for the pq- system.

xp-qx is an axiom, whenever x is composed of hyphens only

5. Write down the three shortest axioms in the pq- system.

-q-p, -q-p-, q-p-

6. Write down the sole rule of production for the pq- system.

Suppose x, y, and z all stand for particular strings containing only hyphens. And Suppose that x py qz is known to be a theorem. The xpy-qz- is a theorem.

7. Show that --p---q----- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

- - p-q - - -

- - p - - q - - - -

- - p - - - q - - - - -

8. Show that -----p----q--------- is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

-----p-q------

- - - - - p - - q - - - - - - -

- - - - - p - - - q - - - - - - - -

9. Write down a string of symbols in the pq- system which is not well formed.

p-p-q

10. State a decision procedure for the pq- system.

The criterion for theorem hood is that the first two
hyphen-groups should add up, in length, to the third hyphen-group.

11. In the longest paragraph on page 48, Hofstadter engages in some "top-down" reasoning. In one sentence, articulate exactly what it is is that he demonstrates with his top-down reasoning in this paragraph?

He demonstrates that any system with only lengthening rules will have a decision procedure for its theorems.

12. In one sentence, characterize "top-down" reasoning.

Working backwards from a string to determine if it is a theorem.

13. In one sentence, characterize "bottom-up" reasoning.

Working from an axiom to determine possible theorems.

14. Consider the procedure for generating theorems of the pq- system given at the top of page 49. What will be in the bucket after executing statement (1a)? After (1b)? After (2a)? After (2b)? After (3a)? After (3b)?

(1a)-p - q - -

(2a)- p - - q - - - | -p - q - -

(3a)-p - – q - - - | - p - - q - - - | -p - q - -

15. What role does the procedure introduced on the top of page 49 play in Hofstadter's presentation of the pq-system and related matters? Answer in just one sentence!

The procedure proves that repeated application of the one rule of the p-q system produces every single possible theorem.

16. What is an isomorphism?

When two complex structures can be mapped onto each other, in such a way that to each part of one structure there is a corresponding part in the other structure, where "corresponding" means that the two part play similar roles in their respective structures.

17. What is an interpretation in the context of a formal system?

Linking symbols to words.

18. When was Linear B deciphered?

When the formal system was successfully interpreted through trial and error.

19. How many meaningful interpretations of the pq- system did Hofstadter present in this chapter.

Two, p as “plus” and q as “equals”

20. How many meaningless presentations of the pq- system are there?

Infinitely many

21. In 50 plus or minus 20 words, summarize what Hofstadter says in the section titled “Formal Systems and Reality”.

A formal system can be used to map processes, like addition being represented by the p-q system. With this in mind, it is theoretically possible to use formal systems to map extremely complex processes in a typographical fashion.