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### Problem Set: The pq-System

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Abstract: This problem set is based on Chapter 2 of Hofstadter's GEB. In it, he presents the first of his Post productions systems that have a numeric flavor. The sequence of production systems that Hofstadter presents, particularly the numeric flavored systems, paves the way for his discussion of Godel's theorem.

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### The 21 Questions and Answers

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1. What is the formal system of Chapter 2 called?

The formal system of chapter 2 is called the pq-System.

2. What are the distinct symbols of this formal system?

The distinct symbols of this system are the p, the q, and the hyphen.

3. How many axioms in the pq- system?

There are an infinite number of axioms in the pq-system.

4. Write down the axiom schema for the pq- system.

$xp-qx-$  is the axiom schema for the pq- system.

# of hyphens + middle # of hyphens = end # of hyphens

5. Write down the three shortest axioms in the pq- system.

$-p-q--$ ,  $--p-q---$ ,  $---p-q----$

6. Write down the sole rule of production for the pq- system.

“Rule: Suppose x, y, and z all stand for particular strings containing only hyphens. And suppose that  $xpyqz$  is known to be a theorem. The  $xpyqz-$  is a theorem.”

7. Show that  $--p---q-----$  is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.  $x = -, y = -, z = --$

- (1)  $--p-q---$  axiom
- (2)  $--p--q----$  by rule 1
- (3)  $--p---q-----$  by rule 1

8. Show that  $----p----q-----$  is a theorem of the pq- system. That is, derive it from an axiom and repeated application of the rule.

- (1)  $----p-q----$  axiom
- (2)  $----p--q-----$  by rule 1
- (3)  $----p---q-----$  by rule 1
- (4)  $----p----q-----$  by rule 1

9. Write down a string of symbols in the pq- system which is not well formed.

$--p--p--p--q-----$

10. State a decision procedure for the pq- system.

The first and second hyphen-group should add up to equal the length of the last hyphen-group

(# of hyphens + middle # of hyphens = end # of hyphens)

11. In the longest paragraph on page 48, Hofstadter engages in some “top-down” reasoning. In one sentence, articulate exactly what it is that he demonstrates with his top-down reasoning in this paragraph?

He demonstrates that there isn't much interest in formal systems that only have lengthening rules. He believes that formal systems are only interesting because they involve lengthening and shortening rules.

12. In one sentence, characterize “top-down” reasoning.

“Top-down” reasoning is working down from specific cases towards basic rules (i.e. ending with the axiom).

13. In one sentence, characterize “bottom-up” reasoning.

“Bottom-down” reasoning is working up from the basics, or axioms, towards specific cases.

14. Consider the procedure for generating theorems of the pq- system given at the top of page 49. What will be in the bucket after executing statement (1a)? After (1b)? After (2a)? After (2b)? After (3a)? After (3b)?

After (1a) -p-q--

After (1b) -p--q---

After (2a) --p-q---

After (2b) --p--q----

After (3a) ---p-q----

After(3b) ---p--q-----

15. What role does the procedure introduced on the top of page 49 play in Hofstadter’s presentation of the pq system and related matters? Answer in just one sentence!

The role the procedure introduced was a “bottom-up” decision procedure

16. What is an isomorphism?

An isomorphism is a mapping between two complex structures in such a way that each part of one structure corresponds to a part in the other structure. If parts “correspond,” this means that they play similar roles in their respective structures.

17. What is an interpretation in the context of a formal system?

An interpretation is a symbol-word correspondence. To interpret a symbol is to assign a meaning to it.

18. When was Linear B deciphered?

Linear B was deciphered between 1951 and 1953 by Michael Ventris and John Chadwick.

19. How many meaningful interpretations of the pq- system did Hofstadter present in this chapter?

There were two meaningful interpretations of the pq-system that Hofstadter presented in the chapter.

20. How many meaningless presentations of the pq- system are there?

There was one meaningless presentation of the pq-system.

21. In 50 +/- 20 words, summarize what Hofstadter says in the section titled “Formal Systems and Reality”.

Formal systems, according to Hofstadter, are analogous to reality because theorems can convey insights about actuality. As a result, Hofstadter asks if all of reality can be expressed by a formal system, and speculates that laws can serve as its production rules. This implies that the axiom of actuality is its configuration at the beginning of time, and that each passing moment of existence is a new theorem formed inside an infinite formal system.