Zach Baker

COG 356 GEB Ch. 7

1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.

P

Q

R

P`

Q`

R`

P``

Q``

R``

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:

(a) How many axioms in the formal system?

None

(b) How many rules in the formal system?

Nine.

(c) What are the names that he gives to these rules?

Joining Rule

Separation Rule

Double-Tilde Rule

Fantasy Rule

Carry-Over Rule

Rule of Detachment

Contrapositive Rule

De Morgan's Rule

Switcheroo Rule

(d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?

Fantasy rule.

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

Joining Rule: If x and y are theorems, then < x ⋀ y > is a theorem

Separation Rule: If < x ⋀ y> is a theorem, then both x and y are theorems.

Double-Tilde Rule: The string '~~' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed.

Fantasy Rule: If y can be derived when x is assumed to be a theorem, then < x ⊃ y > is a theorem.

Carry-Over Rule: Inside a fantasy, any theorem from the “reality “ one level higher can be brought in and used.

Rule of Detachment: If x and < x ⊃ y > are both theorems, then y is a theorem.

Contrapositive Rule: < x ⊃ y > and < ~y⊃~x > are interchangeable.

De Morgan’s Rule: < ~x⋀~y > and ~< x ⋁ y > are interchangeable.

Switcharoo Rule: < x ⋁ y > and <~ x ⊃y > are interchangeable.

4. Derive: <<< P ⋀ Q > ⋀ R > ⊃ < P ⋀ < Q ⋀ R >>>

[	push
<<P ^ Q> ^ R>	premise
<P ^ Q>	separation
R	separation
P	separation
Q	separation
<Q ^ R>	joining
<P ^ <Q ^ R>>	joining
]	pop
<<<P ^ Q > ^ R> ⊃ <P ^ <Q ^ R>>>	fantasy

5. Derive: < < P ∨ Q > ⊃ < Q ∨ P > >

[	push
~<P v Q>	premise
<~P ^ ~Q>	demorgans
~P	separation
~Q	separation
<~Q ^ ~P>	joining
~<Q v P>	demorgans
]	pop
<~<P v Q> ⊃ ~<Q v P>>	fantasy
<<P v Q> ⊃ <Q v P>>	contrapositive

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.

[	push
<~P ^ Q>	premise
~P	separation
~<P v ~Q>	demorgans
<~P ⊃ ~Q>	
<~~P ⊃ ~~Q>	
<P ⊃ Q>	
~Q	

<<P ⊃ Q> ^ ~Q>

]

<<~P ^ Q> ⊃ <<P ⊃ Q> ^ Q>

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

One can use truth tables to make a table, whose columns represent propositions. For each atom, one adds an additional n rows, n being the number of existing atoms, representing the possible combinations of values of the atoms, true or false. One can then determine the truth value of any combination of propositions by adding another column representing it.

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem:<<<P ⊃ Q> ⋀ < ~ P ⊃ Q>> ⊃ Q>

P	Q	P ⊃ Q	~P ⊃ Q	<P ⊃ Q> ^ <~P ⊃ Q>	<<<P ⊃ Q> ⋀ <~P⊃Q>> ⊃ Q>
T	T	T	T	T	T
F	F	T	F	F	T
T	F	F	T	F	T
F	T	T	T	T	T

9. Choose another interpretation for P and Q in Ganto's statement, one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a zentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

P: Q is false

Q: Zach is a good person

If Zach is not a good person, then Zach is a good person, and if Zach is a good person, then Zach is not a good person.

10. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

When conceiving of shortcuts, or “rules about rules,” one loses some of the value of working within a formal system in the first place.

11. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

Even if one were to formalize higher level metatheories, it still would not be sufficient to escape the system.

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

The application of propositional calculus can be expanded upon to include things it was never intended to be used for.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

A proof is a human construct formalized in a human language, while a derivation is a sequence of explicit, trivial steps.

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

Propositional calculus alone can generate contradictions, but when inserted into a larger system which is free of contradictions, it cannot.

15. In one paragraph, write your reaction to this chapter.

I was confused at first, and spent a while generating an answer to question 5, before I realized that basically all of my work was unnecessary. The chapter was nice, I like working with logic. I'm interested in learning more about contradictions and their relationship to the systems which cause them.