

GP - GEB Problem Set: Propositional Calculus

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What's It All About?

This problem set is based on Chapter 7 of Hofstadter's GEB. The problem set features a rather idiosyncratic presentation of the propositional calculus. That said, the presentation nicely contextualizes the propositional calculus within the realms of human reasoning and mathematical reasoning.

Your job is to craft a nicely formatted document consisting of both the questions that you see below and, immediately following the question, your answer to the question.

The Questions

1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.

P, Q, R, P', Q', R', P'', Q'', R''

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:

(a) How many axioms in the formal system?

No axiom, only rules

(b) How many rules in the formal system?

There are 9 rules.

(c) What are the names that he gives to these rules?

Joining, separation, double-tilde, fantasy, carry-over, detachment, contrapositive, de Morgan's and switcheroo rule.

(d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?

The Fantasy Rule

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

Joining Rule: If x and y are theorems, then $\langle x \wedge y \rangle$ is a theorem.

Separation Rule: If $\langle x \wedge y \rangle$ is a theorem, then both x and y are theorems

Double-Tilde Rule: The string ' $\sim\sim$ ' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed

Fantasy Rule: If y can be derived when x is assumed to be a theorem, then $\langle x \supset y \rangle$ is a theorem.

Carry-over Rule: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used.

Rule of Detachment: If x and $\langle x \supset y \rangle$ are both theorems, then y is a theorem.

Contrapositive Rule: $\langle x \supset y \rangle$ and $\langle \sim y \supset \sim x \rangle$ are interchangeable.

De Morgan's Rule: $\langle \sim x \supset \sim y \rangle$ and $\sim \langle x \vee y \rangle$ are interchangeable

Switcheroo Rule: $\langle x \vee y \rangle$ and $\langle \sim x \supset y \rangle$ are interchangeable.

4. Derive: $\langle\langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle \rangle$

[push
$\langle\langle P \wedge Q \rangle \wedge R \rangle$	premise
$\langle P \wedge Q \rangle$	seperation
R	seperation
P	seperation
Q	seperation
$\langle Q \wedge R \rangle$	joining
$\langle P \wedge \langle Q \wedge R \rangle \rangle$	joining
]	pop
$\langle\langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle \rangle$	fantasy rule

5. Derive: $\langle\langle P \vee Q \rangle \supset \langle Q \vee P \rangle \rangle$

[push
$\langle P \wedge Q \rangle$	premise
P	seperation
Q	seperation
$\langle Q \wedge P \rangle$	joining
]	pop
$\langle\langle P \wedge Q \rangle \supset \langle Q \wedge P \rangle \rangle$	fantasy rule

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.

[push
$\langle P \wedge \langle Q \wedge R \rangle \rangle$	premise
P	seperation
$\langle Q \wedge R \rangle$	seperation
Q	seperation
R	seperation
$\langle P \wedge R \rangle$	joining
$\langle \langle P \wedge R \rangle \wedge Q \rangle$	joining
]	pop
$\langle \langle P \wedge \langle Q \wedge R \rangle \rangle \supset \langle \langle P \wedge R \rangle \wedge Q \rangle \rangle$	fantasy rule

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

P	Q	$\langle P \supset Q \rangle$
T	T	T
T	F	F
F	T	T
F	F	T

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem: $\langle\langle P \supset Q \rangle \wedge \langle \neg P \supset Q \rangle \rangle \supset Q$

P	Q	$\neg P$	$\langle P \supset Q \rangle$	$\langle \neg P \supset Q \rangle$	$\langle\langle P \supset Q \rangle \wedge \langle \neg P \supset Q \rangle \rangle$	$\langle\langle\langle P \supset Q \rangle \wedge \langle \neg P \supset Q \rangle \rangle \supset Q \rangle$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

9. Choose another interpretation for P and Q in Ganto's statement, one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

P: I go to class

Q: trap in the elevator

Ganto's statement: If I go to class, I will be trap in the elevator; and if I don't go to class, I will also be trap in the elevator

10. Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

Derived rules are not rules of Propositional Calculus rather they are a part of our knowledge that we have about the system. It is a chain of reasoning that is carried out in the I-mode.

11. Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

Thinking about the system is a way of working in the system. A system's ability to think about itself does not make it outside of itself. An important note is that fallacies can result if we fail to distinguish between working in the system (m-mode) and thinking about the system (i-mode).

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

Propositional calculus can be studied for its own properties, and variants can be made on it. It can also be easily extended to include other fundamental aspects of reasoning.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

A proof is something informal, a product of normal thought written in a human language for human consumption. While a derivation is an artificial version of proof, achieving the same goal through logical structure, with explicit methods that are very simple.

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

Contradiction is a major source of clarification and progress because you start to question your beliefs or modes of reasonings that led to that contradiction.

15. In one paragraph, write your reaction to this chapter.

This chapter was the hardest chapter for me to understand because I associate some of these things with other things, so to relearn each symbol, operation, and rules was very weird. And although some of the symbols, operations and rules were familiar or similar, it was still difficult to think about it in this context. I do really like the stories that were being told throughout the chapter such as the one with Buddha and Ganto's Ax. The stories were funny and made some of it easier to understand and process.