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Chapter 7 Problem Set

1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.

P, Q, R, P', Q', R', P'', Q'', R''

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:

(a) How many axioms in the formal system?

There aren't any axioms in the formal system.

(b) How many rules in the formal system?

There are 9 rules.

(c) What are the names that he gives to these rules?

The names are: the joining rule, the separation rule, the double-tilde rule, the fantasy rule, the carry-over rule, the rule of detachment, the contrapositive rule, De Morgan's rule, and the Switcheroo rule.

(d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?

That rule is the fantasy rule.

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

1.) Joining Rule: If x and y are theorems, $x \wedge y$ is a theorem.

2.) Separation Rule: If $x \wedge y$ is a theorem, then both x and y are theorems

3.) Double-Tilde Rule: The string ' $\sim\sim$ ' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed.

4.) Fantasy Rule: If y can be derived when x is assumed to be a theorem, then $x \supset y$ is a theorem.

5.) Carry-Over Rule: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used.

6.) Rule of Detachment: If x and $x \supset y$ are both theorems, then y is a theorem.

7.) Contrapositive Rule: $\langle \sim y \supset \sim x \rangle$ and $\langle x \supset y \rangle$ are interchangeable.

8.) De Morgan's Rule: $\langle \sim(x \wedge y) \supset \sim x \vee \sim y \rangle$ and $\langle \sim(x \vee y) \supset \sim x \wedge \sim y \rangle$ are interchangeable.

9.) Switcheroo Rule: and $\langle \sim x \supset y \rangle$ are interchangeable.

4. Derive: $\langle \langle \langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle \rangle \rangle$

1.) [(Push)

2.) $\langle \langle P \wedge Q \rangle \wedge R \rangle$ Premise

3.) $\langle P \wedge Q \rangle$ Separation Rule

4.) R Separation Rule

5.) P Separation Rule

6.) Q Separation Rule

7.) $\langle Q \wedge R \rangle$ Joining Rule

8.) $\langle P \wedge \langle Q \wedge R \rangle \rangle$ Joining Rule

9.) [(Pop) 10) $\langle \langle \langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle \rangle \rangle$ Fantasy Rule

5. Derive: $\langle \langle P \vee Q \rangle \supset \langle Q \vee P \rangle \rangle$

1.) [push

2.) $\langle P \vee Q \rangle$ Premise

3.) $\langle \sim P \supset Q \rangle$ Switcheroo Rule

4.) $\langle \sim Q \supset \sim \sim P \rangle$ contrapositive Rule

5.) $\langle \sim Q \supset P \rangle$ Double Tilde Rule

6.) $\langle Q \vee P \rangle$ Switcheroo rule

7.)] Pop Rule

8.) $\langle \langle P \vee Q \rangle \supset \langle Q \vee P \rangle \rangle$ Fantasy Rule

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.

1.) [Push

2.) $\langle P \vee Q \rangle$ Premise

3.) $\langle \sim P \vee Q \sim \rangle$ Double Tilde Rule

4.) $\langle Q \sim \vee P \sim \rangle$ Contrapositive Rule

5.)] Pop Rule

6.) Fantasy Rule

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

Truth tables are used in order to determine true values which can then be used in propositional calculus theorems. This is done by creating a table with all of the true/false values to see what is actually true.

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem:
 $\langle \langle P \supset Q \rangle \wedge \langle \sim P \supset Q \rangle \rangle \supset Q \rangle$

P	Q	$\sim P$	$\langle P \supset Q \rangle$	$\langle \sim P \supset Q \rangle$	$\langle \langle P \supset Q \rangle \wedge \langle \sim P \supset Q \rangle \rangle$	$\langle \langle \langle P \supset Q \rangle \wedge \langle \sim P \supset Q \rangle \rangle \supset Q \rangle$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

9. Choose another interpretation for P and Q in Ganto's statement one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

P: If you see a play Q: I will see a play

If you see a play, I will see a play. If you do not see a play, I will see a play.

10. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

I believe that the most salient idea within the section Shortcuts and Derived Rules is The theorem schema. I believe that because it is a model for other theorem, always leading you to new ones.

11. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

I believe that the most salient idea within the section Formalizing Higher Levels is the fact that you must distinguish between M and I mode always. This way, you create less room in your theorem for fallacies.

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

I believe that the most salient idea within the section Reflections on the Strengths and Weaknesses of the System are the two reasons in which propositional calculus is appealing to mathematicians. It can be studied for its own properties and can be extended to include fundamental aspects of reasoning.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

I believe that the most salient idea within the section Proofs vs Derivations is the note that the rules of propositional calculus cannot be compared to human thought. It is important because as humans we often try to equate non-human things to our own properties.

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

I believe that the most salient idea within the section The Handling of Contradictions is the relationship between clauses and contradictions. I believe that because it is interesting that identifying contradictions works to make a system stronger when able to pinpoint the problem and work towards fixing it.

15. In one paragraph, write your reaction to this chapter.

I found this chapter to be a little difficult at some times, but otherwise it was interesting and worth the read. I am not familiar with calculus at all, having taken the simplest math here at Oswego, but this chapter definitely proved to be useful in making the information on propositional calculus easier for me to understand. I am not sure I still fully grasp it, but hopefully with more practice and review I will feel more comfortable.