GP - GEB Problem Set: Propositional Calculus

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Abstract: This problem set is based on Chapter 7 of Hofstadter's GEB. The problem set features a rather idiosyncratic presentation of the propositional calculus. That said, the presentation nicely contextualizes the propositional calculus within the realms of human reasoning and mathematical reasoning.

1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.

1) P	3) R	5) Q`	7) P``	9) R``
2) Q	4) P`	6) R`	8) Q``	

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:

(a) How many axioms in the formal system?

It doesn't have any.

(b) How many rules in the formals system?

Nine.

(c) What are the names that he gives to these rules?

Rules of inference.

(d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?

The fantasy rule.

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

JOINING RULE: If x and yare theorems, then < xA y> is a theorem.

SEPARATION RULE: If < xA y> is a theorem, then both x and yare theorems.

DOUBLE-TILDE RULE: The string '--' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed.

FANTASY RULE: If y can be derived when x is assumed to be a theorem, then < x::> y> is a theorem.

CARRY-OVER RULE: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used.

RULE OF DETACHMENT: If x and < x:::> y> are both theorems, then y is a theorem.

CONTRAPOSITIVE RULE: < x:::> y> and <- y:::>- x> are interchangeable.

DE MORGAN'S RULE: <- xA- y> and -< xv y> are interchangeable.

SWITCHEROO RULE: < xv y> and <- x::J y> are interchangeable.

4. Derive: <<< P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R >>>

(1) [push

(2) << P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R >> Axiom

(3) < < P ⋀ Q > ⋀ R > Detachment

(4) < P ⋀ Q > Separation

(5) R Separation

(6) Q Separation

(7) P Separation

(8) < P ∧ < Q ∧ R > Detachment

(9) [pop

(10) <<< P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R >>> Fantasy rule

5. Derive: < < P ∨ Q > ⊃ < Q ∨ P > >

(1) [push

(2) < < P ∨ Q > ⊃ < Q ∨ P > > Axiom

(3) < P V Q > Detachment

(4) < ~ P ⊃ Q > Switcheroo

(5) < ~Q ⊃ ~~P > Contrapositive

(6) < Q V ~~ P > Switcheroo

(7) < Q V P > Double-Tilde

(8) [pop

(9) << P V Q > ⊃ < Q V P >> Fantasy Rule

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive, nor Hofstadter derived in his book.

(1) [push

(2) < P ^ Q > Axiom

(3) P Separation

(4) Q Separation

(5) < P V Q > Joining

(6) < ~P > Q > Switcheroo

(7) [pop

(8) < P ^ Q > Fantasy rule

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you

are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether a WFF is a theorem in the propositional calculus.

Working with a given theorem we must break it down and test to see if each element is well formed. One would go about this by creating a truth table with the field of columns containing the symbols and operations of the WFF and the rows containing there correlating True or False values. The next step would be to check if the logic checks out according to the operations and if it does, it’s a well-formed theorem.

8. Using the truth table-based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I’m referring to the section on Gantos Ax. And I’m asking you to show by means of a truth table that the following WFF is a theorem: <<< P ⊃ Q > ∧ < ~ P ⊃ Q > > ⊃ Q >

P	Q	~ P	< P ⊃Q >	< ~ P ⊃ Q > > ⊃ Q >	< P ⊃ Q > ⋀< ~ P ⊃ Q > > ⊃ Q >
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

9. Choose another interpretation for P and Q in Ganto's statement one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

P: If you are sleepy Q: You will be tired

Sentence: If you are sleepy, you will be tired and if you're not sleepy, you will be tired.

10. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

I think the most saliant idea is the 'derived rule' known as the theorem schema. The theorem schema forces you to think in I mode and that's why it's used as a mold for other theorems.

11. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

The most saliant idea in this section is formalizing derived rules as meta theorems for a larger system. This brings up the thought of recursive meta theories for meta theory's and that confuses me.

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

Propositional Calculus is appealing to mathematicians because it can be used to study itself and because it's practical in the proof of reasoning.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

Proofs represent formalizations or hypothesis while derivations are different because it's the logical reasoning behind the proof derived from explicit rules.

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

The most saliant idea in this section is that contradictions are useful to solve issues within the system if properly handled.

15. In one paragraph, write your reaction to this chapter

I found learning about propositional calculus interesting and oddly familiar. Some rules Like the fantasy rules remind me of the time I was first introduced logical rules. This chapter had a lot of content, but I particularly enjoyed the 'Formalizing Higher Levels' section because I had some interesting thoughts about recursive metatheories and meta-metatheories. I really like when Hofstadter says the system can think about itself inside of the system, but it's still not outside itself. Meaning it still can't work outside the system. I thought of Markov processes when I read this section. I don't think I completely understand what Hofstadter was trying to say but it felt like an important statement.