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Propositional Calculus

Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.

1. There are no axioms in Hofstader's presentation. Instead, one uses the fantasy rule. This rule means that one should come up with theorem 'x', and then posit : "what if x were an axiom?" and then use x as the axiom.

Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:
(a)How many axioms in the formal system? -
None.
(b) How many rules in the formals system? -
9
(c) What are the names that he gives to these rules? -
Joining rule
Separation rule
Double-Tilde rule
Fantasy Rule
Carry-Over Rule
Rule of Detachment
De Morgan's Rule
Switcheroo Rule

(d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?

The Fantasy Rule

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

JOINING RULE: If x and y are theorems, then < x∧y> is a theorem.

SEPARATION RULE: If < x∧y> is a theorem, then both x and y are theorems.

DOUBLE-TILDE RULE: The string '~~' can be deleted from any theorem can also be inserted into any theorem, provided that the result string is itself well-formed.

FANTASY RULE: If y can be derived when x is assumed to be a theorem then < x⊃y> is a theorem.

CARRY-OVER RULE: Inside a fantasy, any theorem from the "reality" c level higher can be brought in and used.

RULE OF DETACHMENT: If x and < x⊃y> are both theorems, then y is a theorem.

CONTRAPOSITIVE RULE: <x⊃y> and <~y⊃~x> are interchangeable

DE MORGAN'S RULE: <~x∧~y> and ~< x∨y> are interchangeable.

SWITCHEROO RULE: <x∨y> and <~x⊃y> are interchangeable.

Derive: < < < P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R > > >

1. [push
2. P premise
3. [[push
4. Q premise
5. <P^Q > join
6. [[[push
7. R premise
8.]]] pop
9. <<P^Q>^R> join
10.]] pop
11. <Q^R> join

12. <P^<Q^R>> join
13.] pop
14. **< < < P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R > > >** fantasy rule

Derive: < < P ∨ Q > ⊃ < Q ∨ P > >

1. [push
2. P premise
3. [[push
4. Q premise
5. Q ⊃ P fantasy rule
6. ~P ⊃ ~Q contrapositive
7. ~P detachment
8. <~P ⊃ Q> fantasy rule
9. <<P V Q>> switcheroo
10. [[[push
11. <~P ⊃ ~Q> carry over
12. ~Q detachment
13. << ~Q ⊃ P >> fantasy rule
14. <<Q V P>> switcheroo
15.]]] pop
16.]] pop
17. **< < P ∨ Q > ⊃ < Q ∨ P > >** fantasy
18.] pop

Derive < ~Q ^ ~R>

1. [push
2. Q premise
3. [[push
4. R premise
5. Q ⊃ R fantasy
6. ~Q ⊃ ~R contrapositve

7.	~Q	detachment
8.	[[[	push
9.	~Q ⊃ ~R	carry over
10.	~R	detachment
11.	<~Q ^ ~R>	join
12.	]]]	pop
13.	]]	pop
14.	]	pop

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus. [redacted]

A truth table is a diagram, arranged in rows and columns, that uses true/false values to determine theorems in formal systems. It is most often used in propositional logic. The topmost row dictates the components of the proposed theorem, and the columns dictate true/false values. Working from base components to more complex components as one moves to the right, one may conceptualize it as many if/then statements. If P has a truth value of true, then moving onto the next cell in the row, we may determine if ~P is true, based on the previous cell. This goes for every conceivable truth value in every cell. A proposed string is a theorem, if that theorem's row contains all 'T' truth values.

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm refering to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem: < < < P ⊃ Q > ∧ < ~ P ⊃ Q > > ⊃ Q >

P	Q	<P⊃ Q>	~P	<~P ⊃ Q>	<<P ⊃ Q ^ ~P ⊃ Q> >	<<<P ⊃ Q ^ ~P ⊃ Q>>> ⊃ Q
T	T	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	F	T

9. Choose another interpretation for P and Q in Ganto's statement one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a zentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.

"If you sleep in a bed, you will be rested. If you do not sleep in a bed, you will be rested"

10. **Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.**

A derived rule can basically be said to be a shorthand. If we know a string to be a theorem, then a string that is functionally the same, but perhaps with Q substituted in place of the P, then our urge is to simply write the new string with P, since it is derived the exact same way as the original string. While this is perfectly legitimate, as a derived rule is derived via I-mode, in cannot be included in a formal system, since it is not found within M-mode, and thus cannot be said to be 'formal'.

11. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.**

While it may be tempting to develop a higher system, a 'metasystem', to include derived rules, or 'meta theorems', it would be fallacious. For system A, we may produce metasystem A^N, meaning that we may continue scaling our systems upwards ad infinitum.

12. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.**

While the propositional calculus system is not robust, it is extremely precise and simple. This gives it appeal to fields such as geometry, where simple shapes are discussed. It can also easily be added onto and incorporated into larger and more complex systems.