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GEB Problem Set: Propositional Calculus

Abstract: This problem set is based on Chapter 7 of Hofstadter's GEB. The problem set features a rather idiosyncratic presentation of the propositional calculus. That said, the presentation nicely contextualizes propositional calculus within the realms of human reasoning and mathematical reasoning.

The Questions

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1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.
    a. P
    b. Q
    c. R
    d. R'
    e. Q'
    f. P'
    g. R''
    h. Q''
    i. P''

2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:
    a. How many axioms in the formal system?

        There are three axioms in this formal system (P, Q, and R)

    b. How many rules in the formal system?

        There are ten rules in this formal system

    c. What are the names that he gives to these rules?

        1. Rule of Joining
        2. Rule of Separation
        4. Double-Tilde rule
        5. Fantasy rule
        6. Carry-over rule

7. Rule of detachment
8. Contrapositive rule
9. De Morgan's rule
10. Switcheroo rule

d. What is the one rule that you absolutely must use if you are to derive a theorem in this system?’

   The one rule that is required to be used if one needs to derive a theorem in this system is the rule of joining.

3. Write down each of the rules of the system, just as Hofstadter does on page 187.

- JOINING RULE: If x and y are theorems, then < x ∧ y> is a theorem.
- SEPARATION RULE: If < x ∧ y> is a theorem, then both x and y are theorems.
- DOUBLE-TILDE RULE: The string '~ ~' can be deleted from any theorem and can also be inserted into any theorem, provided that the result string is itself well-formed.
- FANTASY RULE: If y can be derived when x is assumed to be a theorem then < x ⊃ y > is a theorem.
- CARRY-OVER RULE: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used.
- RULE OF DETACHMENT: If x and < x⊃y> are both theorems, then y is a theorem.
- CONTRAPOSITIVE RULE: < ~ x ⊃ ~ y >and < ~ y ⊃ ~ x > are interchangeable
- DE MORGAN'S RULE: <~x∧~y> and ~ < x ∨ y > are interchangeable.
- SWITCHEROO RULE: < x ∨ y> and < ~ x ⊃ y > are interchangeable.

4. Derive: <<< P ∧ Q > ∧ R > ⊃ < P ∧ < Q ∧ R >>>

[	push
< < P ∧ Q > ∧ R >	premise
< P ∧ Q >	separation
P	separation
Q	separation
R	separation
< Q ∧ R >	joining
< P ∧ <Q ∧ R >	joining

] pop

5. Derive: < < P ∨ Q > ⊃ < Q ∨ P > >

[	push
~ < P ∨ Q>	premise
<~ P ∧~Q >	demorgans
~P	separation
~Q	separation
<~Q ∧ ~P>	joining
~<Q ∨ P>	
]	pop
< ~<P ∨ Q> ⊃ ~<Q ∨ P >>	fantasy
< < P ∨ Q > ⊃ <Q ∨ P >>	contrapositive

6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.

7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.

1: Count how many variables you have.
2: Do 2^numberOfVariables. This is going to tell you how many rows your truth table will have
3: Create a column to each variable
4: Now you know the number of rows, make all possible combinations of values True(T) and False(F) using the column of yours variables
5: Look at the operators of your WFF and do the operations:

Negation: (~F) is true if F is false, false if F is true
Conjunction: (F and G) is true if both F and G are true. False otherwise
Disjunction: (F V G) is true if at F or G are true. False otherwise.
Implication: (F -> G) is true unless F is true and G is false
Equivalence: (F <-> G) is true if F and G have the same value

6: A formula F is said to be true under interpretation if F(formula) evaluates to true in the interpretation. Otherwise it is False in the interpretation

8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I’m referring to the **section on Gantos Ax**. And I’m asking you to show by means of a truth table that the following WFF is a theorem: <<< P ⊃ Q > ⋀ < ~ P ⊃ Q > > ⊃ Q >

Truth Table					
P	Q	P⊃ Q	~P⊃Q	<P ⊃ Q> ⋀ <~ P ⊃ Q >	<<< P ⊃ Q > ⋀ < ~ P ⊃ Q > > ⊃ Q >
T	T	T	T	T	T
F	F	T	F	F	T
T	F	F	T	F	T
F	T	T	T	T	T

9. Choose another interpretation for P and Q in Ganto’s statement, one that doesn’t involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto’s statement (what he says to the praying monks) under your interpretation.

P= someone chokes while eating
Q= they die

If someone chokes while eating, they die, and if someone doesn’t choke while eating, they still die.

10. Write down in a meaningful manner, in no more than a few sentences, what you think is the **most salient idea** that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.

Derived rules and shortcuts showcase what has been previously done or assumed in many derivations of theorems, which makes shortcuts to be a perfectly valid procedure when finding new theorems.

11. Write down in a meaningful manner, in no more than a few sentences, what you think is the **most salient idea** that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.

“Even if a system can "think about itself", it still is not outside itself. You, outside the system, perceive it differently from the way it perceives itself. So there still is a metatheory-a view from outside-even for a theory which can "think about itself" inside itself.”

12. Write down in a meaningful manner, in no more than a few sentences, what you think is the **most salient idea** that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.

The simplicity and precision of the Propositional Calculus are exactly the kinds of features which make it appealing to mathematicians, which is why it can be studied for its own properties, exactly as geometry studies simple, rigid shapes and why the Propositional Calculus can easily be extended to include other fundamental aspects of reasoning.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the **most salient idea** that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.

“The Propositional Calculus should be thought of as part of a general method for synthesizing artificial proof-like structures. It does not, however, have much flexibility or generality. It is intended only for use in connection with mathematical concepts-which are themselves quite rigid.”

14. Write down in a meaningful manner, in no more than a few sentences, what you think is the **most salient idea** that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

The contradiction right now of a more relevant example is confronting us-namely the discrepancy between the way we really think, and the way the Propositional Calculus imitates us. This has been a source of discomfort for many logicians, and much creative effort has gone into trying to patch up the Propositional Calculus so that it would not act so stupidly and inflexibly.

15. In one paragraph, write your reaction to this chapter.

It was rather interesting to learn about the complexity, flexibility, and overall the chapter felt unreal and fantasized. Hofstadter showed the propositional calculus to be simple and strong; it can be if you understand how to use and pass these limitations. The decision procedure for theorems was hard to understand and I only half understood it to the point of completing the truth tables (with help).