
Lesson #1: TS in ABC Land

What's It All About?

This little lesson presents a situation with which to think some thoughts that will lead us into the realm of Markov Processes.

The situation

A traveling salesman, TS, sells his products in three cities: A, B and C. If he is in A on a certain day, then he is equally likely to be in any one of the three cities the next day. He never stays in B or C for two days in a row. If he is in B, then he is equally likely to be in one of the other two cities the next day. However, if he is in C then the next day he is twice as likely to be in A as in B. He makes the city of A his permanent residence, and his wife noticed that he will usually start his weekly sales route in this city. As a matter of fact, he is twice as likely to start on Monday in A than B or C, the latter two being equally likely places of his Monday sales. You are to investigate the salesperson's traveling habits!

First questions

1. What is the probability that TS will spend his Monday in city A?
2. What is the probability that TS will spend his Monday in city B?
3. What is the probability that TS will spend his Monday in city C?

An ambiguity

The problem statement includes the following information about the salesman's Monday behavior: "He makes the city of A his permanent residence, and his wife noticed that he will usually start his weekly sales route in this city. As a matter of fact, he is twice as likely to start on Monday in A than B or C, the latter two being equally likely places of his Monday sales." What does this mean? Experience suggests, no matter how much the logical purist might find this offensive, that roughly half of those who interpret this situation choose each of the following interpretations:

1. He is twice as likely to visit A than B and he is twice as likely to visit A than C. (A 50 percent; B 25 percent; C 25 percent)
2. He is twice as likely to visit A than B and C combined. (A 66.67 percent A; B 16.67 percent; C 16.67 percent)

Given that people, generally speaking, don't seem to be able to agree on an interpretation of the given information based solely on language, how might you proceed to confidently know which of the two probabilities reflect reality as you engage in a study of TSs behavior? One possible answer: Perhaps we might further probe his wife.

Reconciling the ambiguity

Given that people, generally speaking, don't seem to be able to agree on an interpretation of the given information based solely on language, how might you proceed to confidently know which of the two probabilities reflect reality as you engage in a study of TSs behavior? One possible answer: Perhaps we might further probe his wife.

We are going to assume that we asked the wife, and that she clarified the matter. Interpretation 1 is the proper interpretation based on her observation of the frequencies with which he visited A and B and C on Mondays. That is, we are going to assume that TS is twice as likely to visit A than B on Monday and he is twice as likely to visit A than C on Monday. (A 50 percent; B 25 percent; C 25 percent)

Exercise #1: Some Probabilities TS in ABC Land

Reasoning in appropriate ways, answer the following 15 questions. Although you are not being asked to turn your work in (this being an exercise), being the good student that you want to be, you should at least compose a document with your answer to each question. Beyond that, you might like to make notes for each question on how you answered the question, or on problems that you perceived as roadblocks to answering the question.

1. Monday (just to be sure we are clear on the interpretation)
 - (a) What is the probability that TS will spend his Monday in A?
 - (b) What is the probability that TS will spend his Monday in B?
 - (c) What is the probability that TS will spend his Monday in C?
2. $A \rightarrow X$
 - (a) Suppose that TS is in A on a given day. What is the probability that TS will spend the next day in A?
 - (b) Suppose that TS is in A on a given day. What is the probability that TS will spend the next day in B?
 - (c) Suppose that TS is in A on a given day. What is the probability that TS will spend the next day in C?
3. $B \rightarrow X$
 - (a) Suppose that TS is in B on a given day. What is the probability that TS will spend the next day in A?
 - (b) Suppose that TS is in B on a given day. What is the probability that TS will spend the next day in B?
 - (c) Suppose that TS is in B on a given day. What is the probability that TS will spend the next day in C?
4. $C \rightarrow X$
 - (a) Suppose that TS is in C on a given day. What is the probability that TS will spend the next day in A?
 - (b) Suppose that TS is in C on a given day. What is the probability that TS will spend the next day in B?
 - (c) Suppose that TS is in C on a given day. What is the probability that TS will spend the next day in C?
5. Tuesday
 - (a) Given all that you know about TS, what is the probability that he will be in A on Tuesday?
 - (b) Given all that you know about TS, what is the probability that he will be in B on Tuesday?
 - (c) Given all that you know about TS, what is the probability that he will be in C on Tuesday?