
GP - GEB Problem Set: Propositional Calculus

What's It All About?

This problem set is based on Chapter 7 of Hofstadter's GEB. The problem set features a rather idiosyncratic presentation of the propositional calculus. That said, the presentation nicely contextualizes the propositional calculus within the realms of human reasoning and mathematical reasoning.

Your job is to craft a nicely formatted document consisting of both the questions that you see below and, immediately following the question, your answer to the question.

The Questions

1. Write down the nine shortest atoms in Hofstadter's presentation of the propositional calculus.
2. Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:
 - (a) How many axioms in the formal system?
 - (b) How many rules in the formal system?
 - (c) What are the names that he gives to these rules?
 - (d) What is the one rule that you absolutely must use if you are to derive a theorem in this system?
3. Write down each of the rules of the system, just as Hofstadter does on page 187.
4. Derive: $\langle \langle \langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle \rangle \rangle$
5. Derive: $\langle \langle P \vee Q \rangle \supset \langle Q \vee P \rangle \rangle$
6. Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.
7. As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.
8. Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Gantos Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem: $\langle \langle \langle P \supset Q \rangle \wedge \langle \sim P \supset Q \rangle \rangle \supset Q \rangle$
9. Choose another interpretation for P and Q in Ganto's statement one that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a sentence corresponding to Ganto's statement (what he says to the praying monks) under your interpretation.
10. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled **Shortcuts and Derived Rules**.
11. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled **Formalizing Higher Levels**.
12. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the **Strengths and Weaknesses of the System**.

13. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled **Proofs vs Derivations**.
14. Write down in a meaningful manner, in no more than a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled **The Handling of Contradictions**.
15. In one paragraph, write your reaction to this chapter.

Further Instructions

Please save your document as a **pdf** file. Only files in the **pdf** format will be accepted. Then, please respond to my email soliciting your work with respect to this assignment, just one time, being sure to attach your **pdf** file. **Please note: This is not an email for you to respond to with questions or comments. Just the pdf file containing your work with respect to this assignment.**

Due date

Monday, April 11, 2022. Any time of the day will do.