GP - GEB Problem Set: The tq- System

What's It All About?

This problem set is based on Chapter 3 of Hofstadter's GEB, in which he introduces the tq- system Just as the pq- system is isomorphic to both addition and subtraction, the tq- system is isomorphic to both multiplication and division. Like the problems posed for the pq- system, these problems focus on the basics of formal systems, such as axioms and theorems and rules of inference for producing theorems.

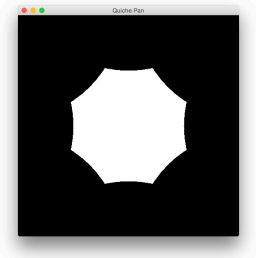
Task

Craft a nicely formatted document consisting of both the questions that you see below, and, immediately following each question, your answer to the question. Please format your work on this problem set in just the same way that you were asked to format the previous Post production system problem sets. And, as always, please save your document as a **pdf** file.

1. Write down the axiom schema and the three shortest axioms in the tq- system.
2. Write down the sole rule of inference for the tq- system and apply it to the well-formed string: `-----t-----q-----`.
3. Reasoning in I-mode, argue that the string you produced in the previous item is not a theorem in the tq- system.
4. Working in M-mode, show that `-----t---q---------------` is a therem in the tq- system.
5. What are the two rules of the C- system?
6. Working within the C- system, argue that `C--------` is a theorem of the system.
7. Does adding the following rule to the C- system constitute a Post production system for determining primes?

   - Suppose x is a hyphen-string. If Cx is **not** a theorem, then Px is a theorem.

   Please explain your rewponse
8. First, please consider the following image of a quiche pan:



   Second, recall that Hofstadter writes the following about positive space and negative space::

   > When a figure or "positive space" (e.g., a human form, or a letter, or a still life) is drawn inside a frame, an unavoidable consequence is that its complementary shape - also called the "ground", or "backgraound", or "negative space" - has also been drawn.

According to this view, the quiche pan shown above, that I computationally rendered, would be considered **negative space**. Explain how this is so. That is, explain how I rendered this image so that the quiche pan may be considered negative space rather than positive space, which would be the normal human interpretation of the image.

9. Consider the A- system as defined by the following axiom and rule:

   - Axiom: `A--`
   - Rule: Suppose that x is a hyphen-string. If `A`x is a theorem, so is `A`x`--`.

   Please answer the following questions with respect to the A-system:

   (a) Show that `A--------` is a theorem of the A- system by working within the system.

   (b) Specify a **decision procedure** for determining theoremhood in the A- system.

   (c) Provide an I-mode argument that the string `A----------` is not a theorem of the A- system.

   (d) What subset of the natural numbers do you think it was my intent to capture with the A- system?

10. Consider the as yet to be formally defined B- system which you should imagine is intended to capture precisely all of the natural numbers that the A- System does not capture.

    (a) Propose, by analogy with the rule on page 66 of GEB, an invalid rule for producing theorems in the B-system.

    (b) Define a (valid) Post produduction system for the B- system in terms of one axiom and one rule.

    (c) Derive `B----------` within the B- system.

    (d) What subset of the natural numbers does the B- system capture?

11. Under **interpretation**, what does the A- system theorem `A--------` say? Under **interpretation**, what does the B- system theorem `B----------` say?

12. According to Hofstadter, what does it mean for a set to be "recursively enumerable"? What does it mean for a set to be "recursive"?

13. Argue that the set of even numbers is **recursively enumerable**.

14. Argue that the set of even numbers is **recursive**.

15. Argue that the set of prime numbers is **recursively enumerable**.

16. Argue that the set of prime numbers is **recursive**.

17. In a sentence or two, explain why you think that I am not asking you in this problem set to derive something like `P-----` within the P- system?

Further Instructions

As previously mentioned, please save your document as a **pdf** file. Only files in the **pdf** format will be accepted. Then, please respond to my email soliciting your work with respect to this assignment, just one time, being sure to attach your **pdf** file. **Please note: This is not an email for you to respond to with questions or comments. Just the pdf file containing your work with respect to this assignment.**

Due date

Friday, February 18, 2022. Any time of the day will do.