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What Is Recursion?

In 1637, the French philosopher Réné Descartes wrote the immortal line "Je pense, donc je suis." Curiously, this is usually rendered in Latin, as *Cogito, ergo sum*, and is translated in English as "I think, therefore I am." In making this statement, Descartes was not merely thinking, he was thinking about thinking, which led him to the conclusion that he existed. The recursive nature of Descartes's insight is perhaps better rendered in the version offered by Ambrose Bierce in *The Devil's Dictionary*: *Cogito cogito ergo cogito sum*—"I think I think, therefore I think I am." Descartes himself, though, was more prone to doubt, and expanded his dictum as "Je doute, donc je pense, donc je suis"—"I doubt, therefore I think, therefore I am." He thus concluded that even if he doubted, someone or something must be doing the doubting, so the very fact that he doubted proved his existence. This probably came as a relief to his friends.

In this book, I examine the more general role of recursion in our mental lives, and argue that it is the primary characteristic that distinguishes the human mind from that of other animals. It underlies our ability not only to reflect upon our own minds, but also to simulate the minds of others. It allows us to travel mentally in time, inserting consciousness of the past or future into present consciousness. Recursion is also the main ingredient distinguishing human language from all other forms of animal communication.

Recursion, though, is a fairly elusive concept, often used in slightly different ways.[1] Before I delve into some of the complexities, let's consider some further examples to give the general idea. First, then, a not-too-serious dictionary definition:

> **Recursion** (rĭ-kûr'-zhən) *noun*. See **recursion.**

One problem here, of course, is that this implies an infinite loop,

Figure 1. The thinker thinks of thinking of thinking (author's drawing).

from which you may never escape in order to read the other stuff in this book. The following variant suggests a way out:

> **Recursion** (rĭ-kûr'-zhən) *noun*. If you still don't get it, see **recursion.**

This banks on the possibility that if you do get it after a round or two, you can escape and move on. If you don't, well I'm sorry.

The postmodern novelist John Barth concocted what is probably both the shortest and the longest story ever written, called *Frame-Tale*. It can be reproduced as follows: Write the sentence *ONCE UPON A TIME THERE* on one side of a strip of paper, and *WAS A STORY THAT BEGAN* on the other side. Then twist one end once and attach it to the other end, to form a Mobius strip. As you work your way round the strip, the story goes on forever.

A similar example comes from an anonymous parody of the first line of Bulwer-Lytton's infamous novel, *Paul Clifford*:

> It was a dark and stormy night, and we said to the captain, "Tell us a story!" And this is the story the captain told: "It was a dark and stormy night, and we said to the captain, 'Tell us a story!' And this is the story the captain told: 'It was a dark . . .' "

Another amusing example is provided by a competition, run by *The Spectator* magazine, which asked readers to state what they

would most like to read on opening the morning paper. The winning entry read as follows:

> Our Second Competition
>
> The First Prize in the second of this year's competitions goes to Mr Arthur Robinson, whose witty entry was easily the best of those we received. His choice of what he would like to read when opening the paper was headed, "Our Second Competition," and was as follows: "The First prize in the second of this year's competitions goes to Mr Arthur Robinson, whose witty entry was easily the best of those we received. His choice of what he would like to read when opening the paper was headed 'Our Second Competition,' but owing to paper restrictions we cannot print all of it."[2]

Taking a different tack, John Barth's story *Autobiography: A Self-recorded Fiction* is a recursive tale in which the narrator is ostensibly the story itself, writing about itself.[3] It ends, recursively, in its own end:

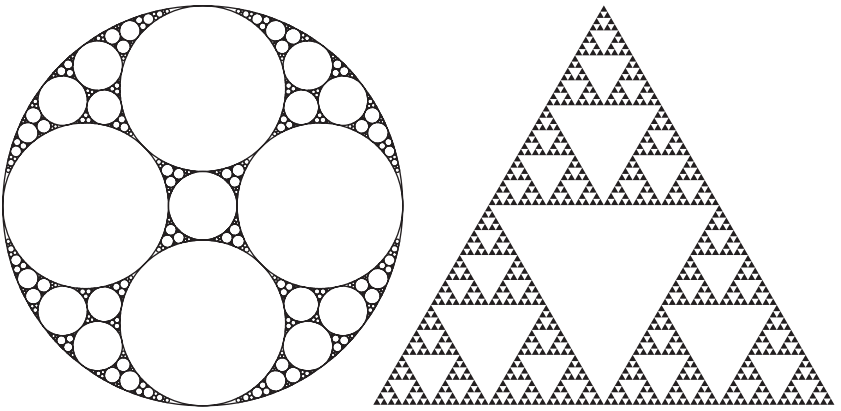
> Nonsense, I'll mutter to the end, one word after another, string the rascals out, mad or not, heard or not, my last words will be my last words.

To my knowledge, no story has yet attempted to write a story of a story that writes about itself.

And then there is the recurring problem of fleas, as penned by the Victorian mathematician Augustus de Morgan:

> Great fleas have little fleas upon their backs to bite 'em,
> And little fleas have smaller fleas, and so *ad infinitum*.
> And the great fleas themselves, in turn, have greater fleas to go on,
> While these again have greater still, and greater still, and so on.[4]

This notion of inserting progressively smaller entities into larger ones *ad infinitum* can also give rise to interesting visual effects, as in the examples shown in figure 2.



Apollonian gasket Sierpinski triangle

Figure 2. Two figures showing recursive insertions of circles (*left*) and triangles (*right*). The Apollonian gasket derives from Apollonius, a Greek geometer from the third century BC, who studied the problem of how to draw a circle that is tangential to three circles. Starting with three circles that are tangential to one another, one can continue the process of constructing circles tangential to all triplets *ad infinitum*. The resulting figure serves as a mathematical model for foam (see Mackenzie 2009 for more information). (2D Apollonian gasket with four initial circles courtesy of Guillaume Jacquenot.)

The use of recursion to create infinite sequences is also exploited by mathematics. One such sequence is the set of natural (i.e. whole) numbers, which I'll write as **N**. Thus we can generate all of the positive natural numbers by the definitions

1 is in **N**
If n is in **N** then (n + 1) is in **N**.

This second definition is recursive, because **N** appears in the condition that needs to be satisfied for **N**.

You may remember, possibly from schooldays, what *factorials* are. As a schoolboy I found them amusing in a childish kind of way, because they were signaled with exclamation marks; thus factorial 3, usually written 3!, is 3 * 2 * 1, and equals 6.[5] Similarly, we can compute the following:

5! = 5 * 4 * 3 * 2 * 1 = 120
8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 40,320

Clearly, this can go on forever, but we can capture the entire set by using just two defining equations:

$$0! = 1$$
$$n! = n * (n - 1)! \quad [\text{where } n > 0].$$

This second equation is recursive in that a factorial is defined in terms of a factorial. We need the first equation to kick the thing off.

The next example is for rabbits, and is called the Fibonacci series, defined by the following three equations:

$$\text{fibonacci}(0) = 1$$
$$\text{fibonacci}(1) = 1$$
$$\text{fibonacci}(n) = \text{fibonacci}(n - 1) + \text{fibonacci}(n - 2)$$
$$[\text{where } n > 1].$$

If you are following me, you should be able to compute the series, which goes 1, 1, 2, 3, 5, 8, 13, . . . What the definition says, then, is that each number in the series is the sum of the two previous ones. Why rabbits? Fibonacci (c. 1170–1250) was an Italian mathematician who used the series to predict the growth of a hypothetical population of rabbits.[6]

For a final informal example, I take you to Kyoto, Japan, where I once happened upon a sign on a gate that was written in Kanji script. I asked what it meant, and my guide told me, I hope correctly, that it meant *Post no bills*. There is a paradox here in that the sign was itself a bill, thereby contravening its own presence. Perhaps there needed to be another sign that said *Post no "Post no bills" bills*. But of course this is itself in violation of its own message, so we might envisage another sign that reads *Post no 'Post "Post no bills" bills' bills*. There is no end to this process, so it might have been more sensible to have allowed bills on the gate in the first place. In practice, though, limitations of time, space, or memory will prevent a recursive sequence of structure continuing forever.

Toward a Working Definition

One of the characteristics of recursion, then, is that it can takes its own output as the next input, a loop that can be extended

indefinitely to create sequences or structures of unbounded length or complexity. In practice, of course, we do not get caught up in infinite loops—life is simply too short for that. For the purposes of this book, then, we shall not be interested so much in the generation of infinite sequences as in a definition that might apply usefully to human thought. A definition that meets this requirement is suggested by Steven Pinker and Ray Jackendoff, who define recursion as "a procedure that calls itself, or . . . a constituent that contains a constituent of the same kind."[7]

The second part of this definition is important, especially in language, because it allows that recursive constructions need not involve the embedding of the *same* constituents, as in the example of the gate in Kyoto, but may contain constituents of the same *kind*—a process sometimes known as "self-similar embedding." For example, noun phrases can be built from noun phrases in recursive fashion. Tecumseh Fitch gives the example of simple noun phrases such as *the dog*, *the cat*, *the tree*, *the lake*, and one can then create new noun phrases by placing the word *beside* between any pair: *the dog beside the tree*, *the cat beside the lake*.[8] Or one might have two sentences: *Jane loves John* and *Jane flies airplanes*, and embed one in the other (with appropriate modification) as *Jane, who flies airplanes, loves John*. These can be extended recursively to whatever level of complexity is desired. For example we could extend the noun phrase to *the dog beside the tree beside the lake*, or the sentence about Jane and John to *Jane who flies airplanes that exceed the sound barrier loves John, who is prone to self-doubt*. Most languages make use of recursive operations of this sort—although we shall see in the next chapter that there may be a few languages that don't operate in this way.

Although it is common to provide illustrations from language, the main theme of this book is that it is in thought rather than in language that recursion originates. As Pinker and Jackendoff put it, "The only reason language needs to be recursive is because its function is to express recursive thoughts. If there were not any recursive thoughts, the means of expression would not need recursion either."[9] In remembering episodes from the past, for instance, we essentially insert sequences of past consciousness into present

consciousness, or in our interactions with other people we may insert what they are thinking into our own thinking. These themes are explored in later chapters.

Process and Structure

As suggested by Pinker and Jackendoff's dual definition, recursion can be understood either as a *process* or as a *structure*. The distinction can be important. A recursive process may lead to a structure that need not be seen as itself recursive. For example, suppose we construct a sequence of musical notes with an embedding routine by pairing pairs of notes, each consisting of a randomly chosen note played on a piano with a randomly chosen note played on a violin. The first pair is embedded in another pair, and the four-note output then embedded in another pair. This process can be continued indefinitely to create a sequence of notes. As illustrated in figure 3, though, the sequence can be interpreted, not as a recursively embedded structure, but as a sequence of piano notes followed by an equally long sequence of violin notes. This failure to distinguish recursive embedding from recursive structure has led to some confusion, especially in claims about recursion in nonhuman species.[10]

Again, in his most recent theory on the nature of language, known as the Minimalist Program,[11] Noam Chomsky has argued that human thought is generated by a Merge operation, applied recursively. That is, units are merged to form larger entities, and the merged entities can be themselves merged to form still larger entities, and so on. This operation underlies the embedded structure of human language, although in Chomsky's theory it applies strictly to what he calls *I-language*, which is the thought process preceding *E-language*, the external language that is actually spoken or signed. Merge can produce strings of elements, be they words or elements of thought, and although it may be applied recursively to produce hierarchical structure, that structure may not be evident in the final output. For instance, even sentences might be regarded simply as words all merged in unstructured sequence, as in ritualized songs or prayers. Everyday language, too, may include mentally undifferentiated clichés and slogans, or sequences that are

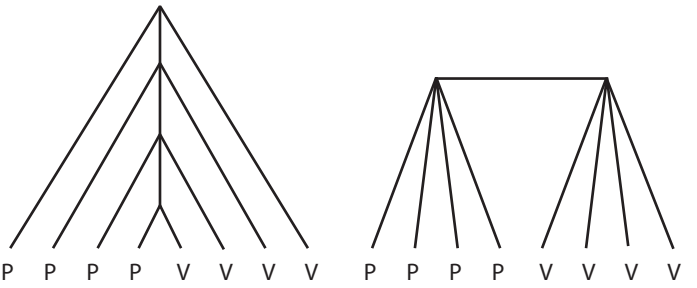


Figure 3. The sequence of Ps and Vs can be created either by recursively nesting PV pairs in PV pairs (*left*), or by arranging a sequence of Ps followed by a sequence of an equal number of Vs (*right*). The sequence might be generated as in the left panel, but interpreted as in the right panel.

highly automated. Politicians may be especially prone to this kind of talk.

As noted above, recursive processes and structures can in principle extend without limit, but are limited in practice. Nevertheless recursion does give rise to the *concept* of infinity, itself perhaps limited to the human imagination. After all, only humans have acquired the ability to count indefinitely, and to understand the nature of infinite series, whereas other species can at best merely estimate quantity, and are accurate only up to some small finite number.[12] Even in language, we understand that a sentence can in principle be extended indefinitely, even though in practice it cannot be—although the novelist Henry James had a damn good try. Such understandings are indeed part of human mental achievement, and depend on a human capacity for recursive thought. Nevertheless they are not the primary concerns of this book.

The appealing aspect of recursion is precisely that it can *in principle* extend indefinitely to create thoughts (and sentences) of whatever complexity is required. The idea has an elegant simplicity, giving rise to what Chomsky called "discrete infinity,"[13] or Wilhelm Humboldt (1767–1835) famously called "the infinite use of finite means." And although recursion is limited in practice, we can nevertheless achieve considerable depths of recursive thought, arguably unsurpassed in any other species. In chess, for example, a player may be able to think recursively three or four steps ahead,

examining possible moves and countermoves, but the number of possibilities soon multiplies beyond the capacity of the mind to hold them.

Deeper levels of recursion may be possible with the aid of writing, or simply with extended time for rehearsal and contemplation, or extended memory capacity through artificial means. The slow development of a complex mathematical proof, for example, may require subtheorems within subtheorems. Plays or novels may involve recursive loops that build slowly—in Shakespeare's *Twelfth Night*, for example, Maria foresees that Sir Toby will eagerly anticipate that Olivia will judge Malvolio absurdly impertinent to suppose that she wishes him to regard himself as her preferred suitor.[14] As in fiction, so in life; we all live in a web of complex recursive relationships, and planning a dinner party may need careful attention to who thinks what of whom.

The structures resulting from recursive processes need not reveal the nature of those processes, just as a loaf of bread may not reveal the processes of kneading that went into the making of the bread, or the taste of wine the picking and trampling of the grapes. Often, though, the structure of a sentence or stream of thought may reveal recursive embedding—interpretation of a sentence may require the understanding of phrases embedded in phrases, regardless of how the embedding was actually accomplished, and an internal understanding of a stream of thought may require the segmentation of episodes within episodes.

What Recursion Is Not

Recursion is not the only device for creating sequences or structures of potentially infinite length or size. I now consider some examples that do not meet the criteria for recursion.

Repetition

Simple repetition can lead to sequences of potentially infinite length, but does not classify as true recursion. For example, the sentence that opens chapter 9 of A. A. Milne's *Winnie the Pooh*

goes *It rained and it rained and it rained*. This could go on forever—or at least until Piglet is drowned—but the repetition simply conveys the information that it rained rather a lot, causing Piglet some ennui. It is not recursive because each addition of *and it rained* is not driven by the previous one; it is simply added at the discretion of the writer.

In any event, repetition does not distinguish human activity from that of nonhuman animals. Birdsong, for example, is relentlessly repetitive, but each repeated theme does not embellish or qualify the previous one. At most, the repetition might signal urgency, or simply signal continuing presence, as one might repeatedly knock on a door in the hope of arousing someone inside. Repetition is ubiquitous in human and animal life, in activities ranging from the repeated jaw movements in eating, to the curiously repetitive nature of sexual activity. The spider, no less, is capable of repetition, as in Walt Whitman's *Leaves of Grass*:[15]

> A NOISELESS, patient spider
> I mark'd, where, on a little promontory, it stood, isolated;
> Mark'd how, to explore the vacant, vast surrounding,
> It launch'd forth filament, filament, filament, out of itself;
> Ever unreeling them—ever tirelessly speeding them

Information can also be aggregated in nonrecursive fashion, as when the short-story writer Saki (H. H. Munro) wrote, "Hunger, fatigue, and despairing hopelessness had numbed his brain."[16] Aggregation of different phrases similarly compounds meaning additively, as when the historian Peter Hennessy wrote:

> The model of a modern Prime Minister would be a kind of grotesque composite freak—someone with the dedication to duty of a Peel, the physical energy of a Gladstone, the detachment of a Salisbury, the balls of a Lloyd George, the word-power of a Churchill, the administrative gifts of an Attlee, the style of a Macmillan, the managerialism of a Heath, and the sleep requirements of a Thatcher.[17]

The sentence itself has recursive elements, but the aggregation of phrases to describe the freakish composite is not recursive in that each does not call the next. Instead, they are effectively elements

in a list, inserted to add information. Nonhuman species may well have a similar ability to accumulate information, as when understanding a predator as large, fierce, and with sharp teeth and claws.

Iteration

A slightly more subtle variant on repetition and aggregation is *iteration*, where a process is repeated, but in this case there is input from the previous application of the process. In this respect it is like recursion, and indeed considered by mathematicians to belong to the class of "general recursive functions." For the main purposes of this book, though, it does not qualify as true recursion because each output is discarded once it has been entered into the next application. The dictionary definition of recursion that I gave earlier in this chapter was also really an example of iteration rather than recursion, because you just keep going round and round the loop, without any added structure. The iterations therefore do not lead to added complexity.[18]

Iterative procedures are used in computational mathematics to arrive at increasingly accurate solutions to a problem. The basic idea is to start with a preliminary solution—perhaps a guess—and then use a procedure to compute a new solution. This solution is then used as the starting point for the next computation, and the new solution is then the starting point for the next round. The cycle is repeated until the solutions stabilize to some acceptable criterion.[19] Feedback systems operate in much the same way, typically as a means of maintaining homeostasis. For example, a thermostat may involve a system for raising or lowering temperature, and the goal is to achieve some given temperature. The actual temperature is fed into the system, which operates to raise or lower the temperature until the desired is reached. The body is awash with feedback systems to maintain homeostasis of temperature, iron, energy, blood composition, and so on. The main regulator is the hypothalamus, in the limbic system of the brain. Such systems again do not differentiate humans from other animals.

Sometimes the distinction between recursion and iteration may be a matter of interpretation. In the infinite loop created by the

parody of *Paul Clifford*, one might say that each beginning of the story is initiated by the previous one, which is then forgotten. The parody is best appreciated, though, if the story is seen as an endless, ever deepening whirlpool, with each segment remaining as part of it. I'm told the story works best if each segment is spoken with a different accent.

Consider too this line from a well-known children's verse:

> This is the dog that worried the cat that killed the rat that ate the malt that lay in the house that Jack built.

To understand this sentence as truly recursive, one must appreciate that it describes a state of affairs as a complex whole, and refers to particular cases of a dog, cat, rat, malt, house, and fellow called Jack. It is not simply the stringing together of a dog that worried a cat, a cat that killed a rat, and so forth. A young child, though, might process it in this piecemeal way, as a succession of unrelated events.

Recursion and Evolutionary Psychology

In emphasizing recursion as a unifying concept, the approach taken in this book contrasts with that adopted by so-called evolutionary psychologists, who have argued that the mind has multiple facets. The basic tenets of evolutionary psychology were laid out in the 1992 volume *The Adapted Mind*, edited by Jerome Barkow, Leda Cosmides, and John Tooby, and popularized by Steven Pinker in his influential 1997 book *How the Mind Works*.[20] Thus Pinker writes that the human mind "is not a single organ but a system of organs, which we can think of as psychological faculties or mental modules."[21] In examining present-day human behavior, the evolutionary psychologist's agenda is to discover independent processes as the basic modules, and relate them to conditions that prevailed in the Pleistocene, when humans existed primarily as hunter-gatherers. As Pinker puts it, the aim is to carve the mind at its joints, so to speak, and "reverse-engineer" its components, or modules, back to the epoch during which the human mind was formed.

In this view, the mind is really a collection of miniminds, each beavering away on its own specific problem, among which are language and theory of mind. This has been called the Swiss-army-knife model of the mind, with a blade for every purpose.[22]

The danger with this approach is that it becomes too easy to postulate modules and to tell "just so" stories about how they evolved, so that there is a risk of returning to the now-abandoned instinct psychology of the early twentieth century.[23] Instinct psychology perished under the sheer weight of numbers—the author of one text counted 1,594 instincts that had been attributed to animals and humans[24]—and evolutionary psychology may also drown in a sea of modules, if not of mixed metaphors. Pinker suggests that we like potato chips because fatty foods were nutritionally valuable during the Pleistocene, but scarce enough that there was no danger of obesity; we like landscapes with trees because trees provides shade and escape from dangerous carnivores on the Africa savanna; flowers please us because they are markers for edible fruits, nuts, or tubers amid the greenery of the savanna; and so on. "There are modules," he writes, "for objects and forces, for animate beings, for minds, and for natural kinds like animals, plants, and minerals."[25]

This is not to say that the Swiss-army-knife model is without merit. Some of the postulated modules do provide insight into the human condition, and are reasonably well founded. For example, foundational work in evolutionary psychology by Leda Cosmides provided good evidence for a "cheater-detection module"—an instinctive ability to detect those who flout social conventions for their own gain.[26] A recent study suggests that humans possess a "category-specific attention system" that is especially adapted for attending to animals;[27] one of the authors, John Tooby, is quoted as saying, "Even dull animals like pigeons . . . recruit a surprising amount of attention—as do turtles resembling rocks."[28] This book is not intended to deny that there are many specific dispositions that shape our mental and social lives; rather, my aim is to suggest that there are deeper aspects of human thought that are governed by similar principles, and that recursion is one of those principles—and perhaps the most important one.

To be fair, too, not all evolutionary psychologists have insisted that modules are completely encapsulated, shut off from any communication with one another. Even Steven Pinker, for example, writes, "[Modules] accomplish specialized functions, thanks to their specialized structures, but don't necessarily come in encapsulated packages."[29] Steven Mithen, although scarcely a card-carrying evolutionary psychologist, argued that the human mind evolved its distinctive character precisely because previously encapsulated modules began to "leak," creating what he calls "cognitive fluidity."[30] It is as though the modules stopped minding their own businesses, and began to gossip. My approach in this book is not entirely at odds with this view, in that I argue a common principle might underlie a number of our distinctive abilities.

Others are also beginning to question the Swiss-army-knife model of the human mind more starkly. David Premack, for example, adopts an approach similar to that offered in this book. Reviewing the evidence for discontinuity between humans and other animals, he writes: "Animal competencies are mainly adaptations restricted to a single goal. Human competencies are domain-general and serve numerous goals."[31] This in effect reverses the evolutionary psychology argument—the mind has become less rather than more modular. The tide may well be turning.

In any event it is unlikely that recursion can be considered a module. As we shall see, recursion seems to be an organizing principle in very different spheres of human mental activity, from language to memory to mind reading. Recursive thinking probably depends on other mental attributes. One of these is what has been termed working memory, which holds information in consciousness. In order to embed processes within processes it is necessary to remember where one had got to in the earlier process when an embedded process has been completed. For instance, in a sentence like *My dog, who eats bananas, often gets sick*, one must hold the early part of the sentence (*My dog*) and link it to the next part (*often gets sick*). Dwight W. Read has argued that nonhuman primates, even our closest relatives the chimpanzees, have a working memory that is too limited to allow this kind of embedding.[32] Recursion probably also depends on an executive process that or-

ganizes what is to be embedded in what, and this may depend on the frontal lobes of the brain. The ability to organize and carry out recursive operations may therefore depend on several processes.

Although I do not embrace the modular view assumed by evolutionary psychologists, I am at one with them in proposing that a distinctively human mind evolved during the Pleistocene, the epoch that stretched from around 2.6 million years ago to some 12,000 years ago. How that happened will be told in the later chapters of this book.

Plan of the Book

The book is divided into four parts.

Part 1 deals with language. Although recursion is not limited to language, it is most commonly invoked to explain why human language differs from other forms of animal communication, an insight largely attributable to Noam Chomsky. Chapter 2 discusses the nature of language, with particular emphasis on the role of recursion. Chapter 3 then raises the age-old question of whether other animals have anything resembling human language. Chapter 4 develops the idea that language evolved from manual gestures—an idea that suggests greater evolutionary continuity between humans and other primates than the more common assumption that language emerged from vocal calls.

Part 2 deals with mental time travel, the ability to bring to mind events removed from the present in both time and place. Chapter 5 starts with memory, and develops the idea that memory for specific episodes is unique to humans. Chapter 6 extends the notion of episodic memory to the imagining of possible future events, leading to the concept of the self as existing through time. This leads to the notion, discussed in chapter 7, that language itself evolved to enable people to share their memories and plans, and so to communicate about events that are not present in the immediate environment. This leads also to fiction—the telling of stories that need not be based on fact, but that nonetheless hone the capacity to deal with the episodic exigencies of human social life.

Part 3 deals another recursive aspect of human thought, namely, theory of mind—or the ability to understand what others are thinking or feeling. Chapter 8 introduces mind-reading, not as a psychic phenomenon, but as a natural ability to infer the mental perspectives of other people. This ability is again critical to social cohesion and cooperation. Chapter 9 explains how theory of mind was also critical to the emergence of language.

Part 4 delves more specifically into the question of how the recursive mind evolved. Chapter 10 sets this question in the context of the classic debate between Cartesian discontinuity and Darwinian continuity. Chapter 11 examines some of the steps by which the hominins,[33] after splitting from the line leading to modern chimpanzees and bonobos, began to assume human-like attributes. Chapter 12 then considers the final step to "modern" *Homo sapiens*, the sole surviving hominin species—dominant, manipulative, Machiavellian, and capable of pondering our own nature and status on the planet. That, perhaps, is the ultimate triumph of the recursive mind.

Chapter 13 presents the final summary and conclusions.